Study Material

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B.Sc. I (Hons)

Paper : 1.

Topic : Solution of Cubic Equation by Cardan's method.

Material Sl. no. — 2.

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Solution of cubic Equation by Cardan's Method

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Method of Solution

Let a cubic equation be

$$a_0x^3 + 3a_1x^2 + 3a_2x + a_3 = 0 \quad \text{—— (1)}$$

This can be reduced to its standard form.

Let it reduced to $z^3 + 3Hz + G = 0$ — (2)

Where $z = a_0x + a_1$, $H = a_0a_2 - a_1^2$ and

$G = a_0^2a_3 - 3a_0a_1a_2 + 2a_1^2$

Firstly let us put $z = u + v$.

Then, $z^3 = (u+v)^3$

$$= u^3 + v^3 + 3uv(u+v)$$

$$= u^3 + v^3 + 3uvz$$

or, $z^3 - 3uvz - (u^3+v^3) = 0$ ..... (3)

Comparing (2) and (3) we get

$uv = -H$

& $u^3 + v^3 = -G$. ——— (4)

Now $(u^3 - v^3)^2 = (u^3+v^3)^2 - 4u^3v^3$

$$= G^2 - 4(-H)^3$$

$$= G^2 + 4H^3$$

or $u^3 - v^3 = \sqrt{G^2 + 4H^3}$ ——— (5)

Adding ④ & ⑤ we get

$$2u^3 = -G + \sqrt{G^2 + 4H^3}$$

$$\text{or, } u^3 = \frac{-G + \sqrt{G^2+4H^3}}{2}$$

Again ④ − ⑤ gives.

$$2v^3 = -G - \sqrt{G^2+4H^3}$$

$$\text{or } v^3 = \frac{-G - \sqrt{G^2+4H^3}}{2}.$$

Let r be one of the value of $\left[\frac{-G+\sqrt{G^2+4H^3}}{2}\right]^{1/3}$ i.e. u Then ~~other~~ all values of p are given by $r, r\omega, r\omega^2$, [ω is imaginary cube root of unity]

As $uv = -H$.

So the values of v are given below respectively

When $u = r$, $v = -H/r$

When $u = r\omega$, $v = -\frac{H}{r\omega} = -\frac{H\omega^2}{r}$

When $u = r\omega^2$, $v = -\frac{H}{r\omega^2} = -\frac{H\omega}{r}$.

So the values of z are $r - H/r$, $r\omega - \frac{H\omega^2}{r}$, $r\omega^2 - \frac{H\omega}{r}$.

Hence the values of x are given as below

$$\frac{1}{a_0}\left(r - H/r - a_1\right),\ \frac{1}{a_0}\left(r\omega - \frac{H\omega^2}{r} - a_1\right),\ \frac{1}{a_0}\left(r\omega^2 - \frac{H\omega}{r} - a_1\right)$$

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Worked out Example

Solve by Cardans method

1. $x^3 - 27x - 54 = 0$

Soln Given equation is

$$x^3 - 27x - 54 = 0 \quad \text{——— (i)}$$

As it is in standard form. So

Let $x = u + v$

or $x^3 = u^3 + v^3 + 3uvx$

or, $x^3 - 3uvx - (u^3 + v^3) = 0 \quad \text{——— (ii)}$

Comparing (i) & (ii) we get

$uv = 9 \quad = -H$

$u^3 + v^3 = 54 \quad = -G$

So, $u^3 = \frac{1}{2}\left[54 + \sqrt{54^2 + 4(-9)^3}\right]$

$= \frac{1}{2}\left[54 + 0\right]$

$u^3 = 27$

or, $u = (27)^{1/3}$

So values u are $3, 3\omega, 3\omega^2$, [ω is one of the imaginary cube root of unity]

Corresponding values of v are $3, 3\omega^2, 3\omega$

Hence solution of the equation (i) are given by:

$6, \ 3(\omega + \omega^2), 3(\omega^2 + \omega)$

i.e, $6, -3, -3$.