Material Sl. no. 34

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Pedal Equation.

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Definition

When the equation of a curve expressed in terms of (p, r), then form is called pedal equation of that curve, where p is the perpendicular distance from the pole to the tangent and r is the radius vector.

How to find the pedal equation of a curve from its polar equation

Let the equation of the curve in polar form be

$$f(r, \theta) = 0 \quad \text{——— (1)}$$

We know that

$$\tan\phi = r\frac{d\theta}{dr} \quad \text{——— (2)}$$

&

$$p = r\sin\phi \quad \text{——— (3)}$$

Where ϕ is the angle between tangent and radius vector.

Eliminating θ from ① ②, ② & ③ we get the pedal equation of the curve.

How to find the pedal equation of a curve from its cartesian equation

Let the equation of a curve in cartesian form be

$$f(x, y) = 0. \quad \text{---------- ①}$$

So equation of tangent at any point (x, y) is

$$Y - y = \frac{dy}{dx}(X - x)$$

or, $$Y - \left(\frac{dy}{dx}\right)X + x\frac{dy}{dx} - y = 0.$$

So clearly length of perpendicular from origin (pole)

$$p = \frac{x\frac{dy}{dx} - y}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}} \quad \text{---------- ②}$$

Again we have $r^2 = x^2 + y^2$ ---------- ③

Eliminating x & y from ①, ② & ③ we get pedal equation of the curve.

Example.

1. Find the pedal equation of the curve $r^m = a^m \cos m\theta$.

Soln. : Given $r^m = a^m \cos m\theta$ ——— ①

Taking logarithm we get

$$m \log r = \log a^m \cos m\theta$$

$$= \log a^m + \log \cos m\theta$$

$$m \log r = m \log a + \log \cos m\theta$$

Differentiating above w.r.t. θ we get,

$$\frac{m}{r} \frac{dr}{d\theta} = \frac{-m \sin m\theta}{\cos m\theta}$$

$$\text{or, } \frac{1}{r} \frac{dr}{d\theta} = -\tan m\theta.$$

$$\text{or, } r\frac{d\theta}{dr} = -\cot m\theta = \tan\left(\frac{\pi}{2} + m\theta\right)$$

$$\text{or, } \tan\phi = \tan(\pi/2 + m\theta)$$

$$\text{or, } \phi = \pi/2 + m\theta$$

Again we know that

$$p = r\sin\phi$$

$$= r\sin(\pi/2 + m\theta)$$

$$\text{or, } p = r\cos m\theta$$

$$\text{or, } \cos m\theta = p/r.$$

Putting this value of $\cos m\theta$ in equation (1) we get

$$r^m = a^m \cdot \frac{p}{r}$$

$$\text{or, } p = \frac{r^{m+1}}{a^m}$$

Which is the required Polar equation.