Material Sl. no. 37

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Curvature

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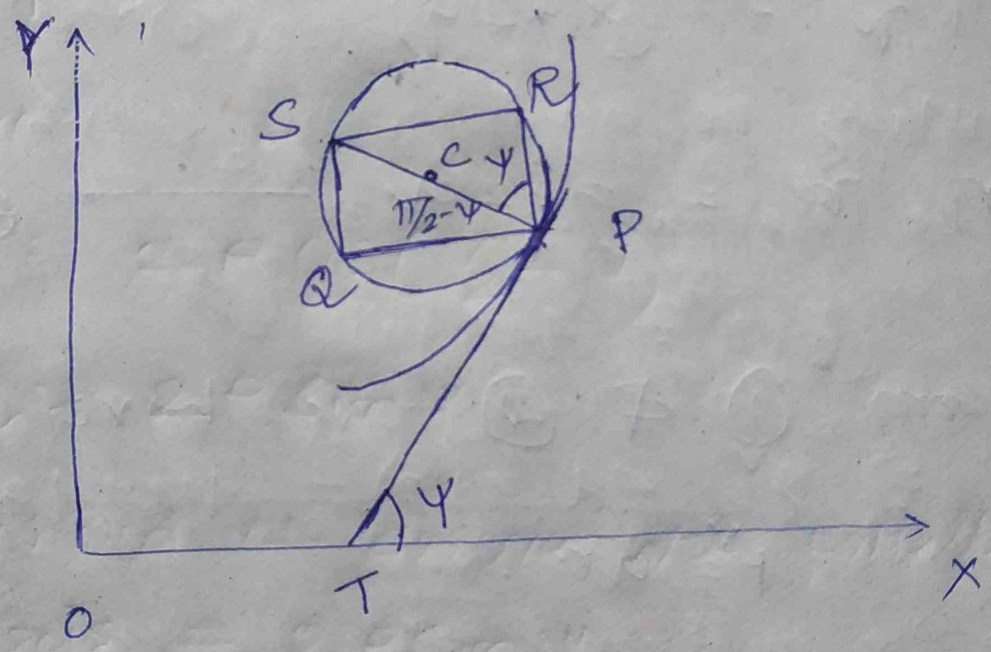
Chord of Curvature.

Chord of curvature parallel to the co-ordinate axes:



Let $y = f(x)$ be the equation of the curve and P be any point on the curve.

Let QPR be the circle of curvature at P with centre at C.

We now draw chord PQ parallel to x - axis and chord PR parallel to y - axis.

Let us draw QRS from R parallel to PQ and QS parallel to PR.

Now we draw tangent at P to the curve and let it meet the x - axis at T.

Let $\angle PTX = \psi$

So clearly $\angle QPT = \psi$.

As. PQ is parallel to OX.

Now $\angle SPT = \pi/2$.

Hence $\angle SPQ = \pi/2 - \psi$.

So $\angle SPR = \psi$

We know that semi circular angle is right angle.

So $\angle SQP = \pi/2$

From the triangle ΔPQS we have

$$PQ = PS \cos \angle SPQ$$
$$= PS \cos(\pi/2 - \psi)$$

Now if ~~we~~ let ρ is the radius of the circle i.e $CP = \rho$

Then clearly $SP = 2\rho$.

So we have

$$PQ = 2\rho \cos(\pi/2 - \psi)$$

$$PQ = 2\rho \sin\psi$$

Hence, chord of curvature parallel to x-axis $= 2\rho \sin\psi$.

Again since $\angle PRS = \pi/2$

Hence from the triangle ΔPRS we have, $PR = PD \cos\angle SPR$

$PR = 2P \cos\psi$

$\therefore$ chord of curvature parallel to y-axis $= 2P \cos\psi$.