Material Sl. no. 41

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Curvature

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Example

Find the co-ordinates of the centre of the curvature for the curve $y^2 = 4ax$.

Soln. Equation of the given curve $y^2 = 4ax$

Differentiating we get.

$$2y y_1 = 4a$$

$$\text{or, } y_1 = \frac{2a}{y}$$

Again differentiating we get

$$y_2 = -\frac{2a}{y^2} \cdot y_1$$

$$= -\frac{2a}{y^2} \cdot \frac{2a}{y}$$

$$= \frac{-4a^2}{y^3}$$

We know that if (α, β) be the centre of curvature, then.

$$\alpha = x - \frac{y_1(1+y_1^2)}{y_2} \quad \&.$$

$$\beta = y + \frac{1+y_1^2}{y_2}$$

So, $$\alpha = x - \frac{y_1(1+y_1^2)}{y_2}$$

$$= x - \frac{\frac{2a}{y}\left\{1+\left(\frac{2a}{y}\right)^2\right\}}{-\frac{4a^2}{y^3}}$$

$$= x + \frac{\frac{2a}{y}\left(1+\frac{4a^2}{y^2}\right)}{\frac{4a^2}{y^3}}$$

$$= x + \frac{2a}{y} \cdot \frac{(y^2+4a^2)}{y^2} \cdot \frac{y^3}{4a^2}$$

$$= x + \frac{y^2+4a^2}{2a}$$

$$= x + \frac{4ax+4a^2}{2a}$$

$$= x + \frac{4a(x+a)}{2a}$$

$$= x + 2(x+a)$$

$$= 3x+2a$$

Again $$\beta = y + \frac{1+y_1^2}{y_2}$$

$$= y + \frac{1+\left(\frac{2a}{y}\right)^2}{-\frac{4a^2}{y^3}}$$

$$= y - \frac{1+\frac{4a^2}{y^2}}{4a^2/y^3}$$

$$= y - \frac{y^2 + 4a^2}{y^2} \cdot \frac{y^3}{4a^2}$$

$$= y - \frac{y}{4a^2}(y^2 + 4a^2)$$

$$= y - \frac{y}{4a^2}(4ax + 4a^2)$$

$$= y - \frac{y}{a}(x + a)$$

$$= y\left(1 - \frac{x+a}{a}\right)$$

$$= y\left(\frac{a - x - a}{a}\right)$$

$$= -\frac{xy}{a}$$

$$= -\frac{x\sqrt{4ax}}{a}$$

$$= -\frac{2}{\sqrt{a}} x^{3/2}.$$

So, centre of the curvature is given by

$$\left(3x + 2a, -\frac{2}{\sqrt{a}} x^{3/2}\right)$$