Material Sl. No. 36

Pedal Equation

Example

Find the pedal equation of the the parabola $y^2 = 4ax$ w.r.t. its focus.

Soln

Equation of the parabola is $y^2 = 4ax$

Shifting the origin to the focus, the eqn of the parabola becomes

$$(Y+0)^2 = 4a(x+a)$$

$$\Rightarrow Y^2 = 4a(x+a) = 4ax + 4a^2$$

is the equation of the tangent at any point (x, Y) is given by

$$YY = 2a(X+x) + 4a^2$$

or, $YY = 2aX + 2ax + 4a^2$

or $2aX - YY + 2ax + 4a^2 = 0$

So length of perpendicular from the origin to the tangent =

$$p = \frac{4a^2 + 2ax}{\sqrt{4a^2 + y^2}}$$

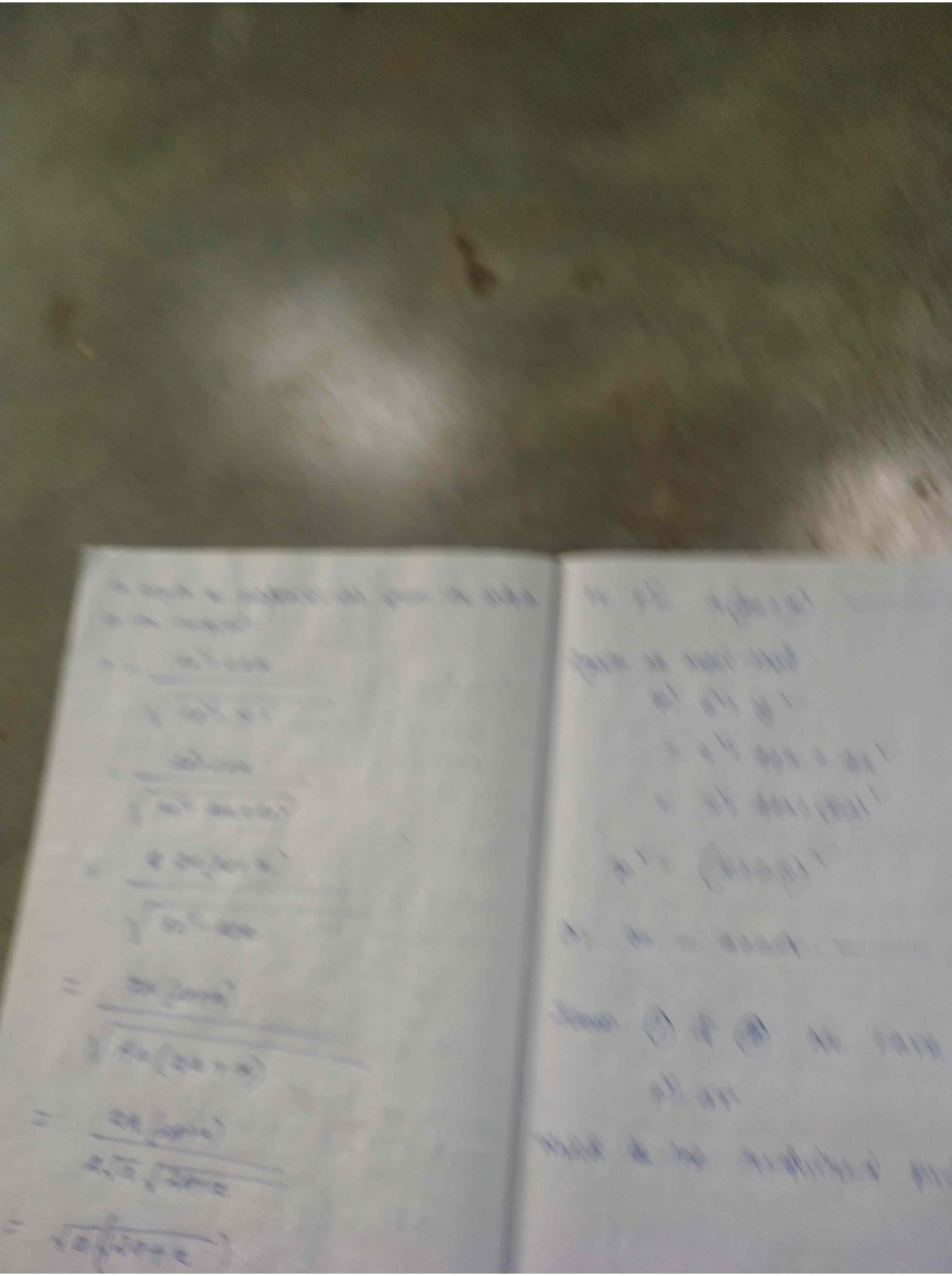
$$= \frac{4a^2 + 2ax}{\sqrt{4a^2 + 4ax + 4a^2}}$$

$$= \frac{2a(2a + x)}{\sqrt{8a^2 + 4ax}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{2\sqrt{a}\sqrt{2a + x}}$$

$$= \sqrt{a}\left(\sqrt{2a + x}\right)$$



or, $p^2 = a(2a+x)$ ——— (1)

Again we know that.

$$r^2 = x^2 + y^2$$

$$= x^2 + 4ax + 4a^2$$

$$= x^2 + 4ax + (2a)^2$$

$$r^2 = (x+2a)^2$$

or, $r = x+2a$. ——— (2)

From (1) & (2) we have

$$p^2 = ar$$

which is the required pedal equation.