

Material Sl. no. 36

 * *
Pedal Equation
 * *

~~Problem~~ Example

Find the pedal equation of the the parabola $y^2 = 4ax$ w.r.t. its focus.

Soln

Equation of the parabola is $y^2 = 4ax$

Shifting the origin to the focus, the ~~to~~ eqn of the parabola becomes

$$(y+0)^2 = 4a(x+a)$$

$$\Rightarrow y^2 = 4a(x+a) = 4ax + 4a^2$$

So the equation of the tangent at any point (x, y) is given by

$$Yy = 2a(x+x) + 4a^2$$

$$\text{or, } Yy = 2ax + 2ax + 4a^2$$

$$\text{or } 2ax - Yy + 2ax + 4a^2 = 0$$

So length of perpendicular from the origin to the tangent =

$$p = \frac{4a^2 + 2ax}{\sqrt{4a^2 + 4ax}}$$

$$= \frac{4a^2 + 2ax}{\sqrt{4a^2 + 4ax}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a^2 + 4ax}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \frac{2a(2a + x)}{\sqrt{4a(2a + x)}}$$

$$= \sqrt{a}(\sqrt{2a + x})$$

$$\begin{aligned}
 &= \frac{2x(200x)}{\sqrt{40(200x)}} \\
 &= \frac{2x(200x)}{2\sqrt{10(200x)}} \\
 &= \frac{2x(200x)}{2\sqrt{2000x}} \\
 &= \frac{2x(200x)}{2\sqrt{2000} \sqrt{x}} \\
 &= \frac{2x(200x)}{2\sqrt{2000} \sqrt{x}} \\
 &= \frac{2x(200x)}{2\sqrt{2000} \sqrt{x}} \\
 &= \frac{2x(200x)}{2\sqrt{2000} \sqrt{x}}
 \end{aligned}$$

The first part of the problem is to find the area of the square. The side length of the square is given as 200x. The area of a square is given by the formula $A = s^2$, where s is the side length. Therefore, the area of the square is $(200x)^2 = 40000x^2$.

The second part of the problem is to find the perimeter of the square. The perimeter of a square is given by the formula $P = 4s$, where s is the side length. Therefore, the perimeter of the square is $4(200x) = 800x$.

The third part of the problem is to find the area of the circle. The radius of the circle is given as $\sqrt{10x}$. The area of a circle is given by the formula $A = \pi r^2$, where r is the radius. Therefore, the area of the circle is $\pi(\sqrt{10x})^2 = 10\pi x$.

The fourth part of the problem is to find the area of the square minus the area of the circle. This is given by $40000x^2 - 10\pi x$.

$$\text{or, } p^{\vee} = a(2a + x) \quad \text{--- (1)}$$

Again we know that

$$r^{\vee} = a^{\vee} + y^{\vee}$$

$$= 2^{\vee} + 4ax + 4a^{\vee}$$

$$= 2^{\vee} + 4ax + (2a)^{\vee}$$

$$r^{\vee} = (x + 2a)^{\vee}$$

$$\text{or, } r = x + 2a \quad \text{--- (2)}$$

From (1) & (2) we have

$$p^{\vee} = ar$$

which is the required pedal equation.