# Numerical Analysis.

**Note:** The N-R formulae be

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{------} (i)$$

The iteration formulae be $x_{n+1} = \phi(x_n)$ --- (ii)

The inner formulae (i) shows that it is an iteration process of the form $x = \phi(x)$, where

$$\phi(x) = x - \frac{f(x)}{f'(x)}.$$

$\therefore$ N-R method converge when $|\phi'(x)|$

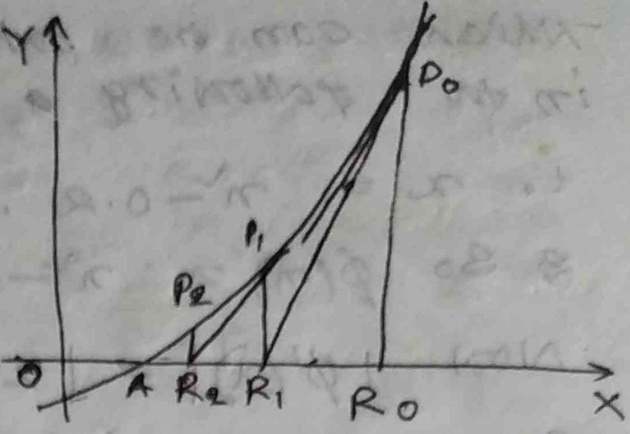
$$= \left| \frac{d}{dx}\left\{ x - \frac{f(x)}{f'(x)} \right\} \right| < 1$$

i.e $$\left| 1 - \frac{\{f'(x)\}^2 - f(x) f''(x)}{\{f'(x)\}^2} \right| < 1$$

$$= \left| \frac{f(x) f''(x)}{\{f'(x)\}^2} \right| < 1$$

i.e. $|f(\alpha) f''(\alpha)| < \{f'(\alpha)\}^2$.

## Geometrical derivation of N-R method:




At the given figure represents the graph of the function $y = f(x)$ in the n.b.d of the point A, where it crosses the x-axis.

Let $\alpha_0$ be the initial starting point.

Now to calculate the next approximation of the root we are to replace the graph at $P_0(\alpha_0, f(\alpha_0))$ by the tangent to the curve at $P_0$. If this tangent meets the x-axis at $R_1$ then.

$$\alpha_1 = OR_1$$
$$= OR_0 - R_1R_0$$
$$= \alpha_0 - \frac{f(\alpha_0)}{f'(\alpha_0)}$$

From $\Delta P_0R_1R_0$,

$$R_1R_0 = \frac{P_0R_0 \cdot R_1R_0}{P_0R_0}$$
$$= f(\alpha_0) \cdot \frac{1}{\tan \angle P_0R_1R_0}$$
$$= \cancel{f(\alpha_0) \cot \angle P_0R_1R_0}$$
$$= \frac{f(\alpha_0)}{f'(\alpha_0)}$$

Similarly to get the 2nd approximation $\alpha_2$ we are to replace the graph at $P_1(\alpha_1, f(\alpha_1))$ by the tangent to the curve at $P_1$ and we get the approximate value of the root as.

$$\alpha_2 = \alpha_1 - \frac{f(\alpha_1)}{f'(\alpha_1)}$$

Proceeding in this way, we get.

$$\alpha_{n+1} = \alpha_n - \frac{f(\alpha_n)}{f'(\alpha_n)}$$

Express $x^2 - x - 0.2 = 0$ in the form $x = \phi(x)$ in different ways and pick out the possible form to be used to solve this equation by integration de method for the real root lying between

1 and 2 and solve it.

: Let $f(x) \equiv x^2 - x - 0.2 = 0$ which can be put in the form $x = \phi(x)$ in the following 2 ways:

1. $x = x^2 - 0.2$.

So $\phi(x) = x^2 - 0.2$.

Now $|\phi'(x)| = |2x| \nless 1$ in $(1, 2)$.

So above form of $x = \phi(x)$ is rejected.

2. $x^2 - x - 0.2 = 0$

or, $x^2 = x + 0.2$

or, $x = \sqrt{x + 0.2}$.

Here $\phi(x) = \sqrt{x + 0.2}$

$\therefore |\phi'(x)| = \left|\frac{1}{2\sqrt{x+0.2}}\right| < 1$. for $\forall x$ in $(1, 2)$.

This form of $x = \phi(x)$ will be useful for iteration method.

Let the initial approximation be $\alpha_0 = 1$. (since the root lies in between 1 and 2).

Then the successive approximations of the desired root are as below:

$\alpha_1 = \phi(\alpha_0) = \phi(1) = \sqrt{1 + 0.2} = \sqrt{1.2}$

$\simeq 1.095$

$\alpha_2 = \phi(\alpha_1) = \phi(1.095) = \sqrt{2.095} = 1.378$

$\alpha_3 = \phi(\alpha_2) = \phi(1.378) = \sqrt{2.378} = 1.157$

$\alpha_4 = \phi(\alpha_3) = \phi(1.157) = 1.165$

$\alpha_5 = \phi(\alpha_4) = \phi(1.165) = 1.168$

So the root $= 1.17$ (correct upto significant figures).

3. $x^2 - x - 0.2 = 0$

or, $x(x-1) = 0.2$

or, $x = \frac{0.2}{x-1}$

Therefore $\phi(x) = \frac{0.2}{x-1}$.

Now $|\phi'(x)| = \left|\frac{-0.2}{(x-1)^2}\right| \not< 1$ in $(1,2)$

~~Also~~.

4. $x - 1 = \frac{0.2}{x}$.

or, $x = 1 + \frac{0.2}{x}$.

$\therefore \phi(x) = 1 + \frac{0.2}{x}$.

$|\phi'(x)| = \left|\frac{-0.2}{x^2}\right| < 1$ $(1,2)$.

Proceeding as above as in form 2, we can have the root 1.17 (correct upto three significant figures).