For- B.Sc. I (Math Hons)
Paper - 1

Relation between roots and co-efficients

Let $f(x) = a_0x^n + a_1x^{n-1} + \cdots + a_n$ be a polynomial of degree n, $(a_0 \neq 0)$.

Let $\alpha_1, \alpha_2, \cdots, \alpha_n$ be the roots of the equation $f(x) = 0$

Then we have the identity

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$$

$$= a_0(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)\cdots(x-\alpha_{n-1})(x-\alpha_n)$$

$$= a_0\{x^n - (\alpha_1+\alpha_2+\alpha_3+\cdots+\alpha_n)x^{n-1} + (\alpha_1\alpha_2+\alpha_1\alpha_3+\cdots+\alpha_{n-1}\alpha_n)x^{n-2} + \cdots + (-1)^n\alpha_1\alpha_2\alpha_3\cdots\alpha_n\}$$

$$= a_0\{x^n - \Sigma\alpha_1 x^{n-1} + \Sigma\alpha_1\alpha_2 x^{n-2} + \cdots + (-1)^n\alpha_1\alpha_2\cdots\alpha_n\}$$

From the equality of polynomials it follows that

$$a_1 = a_0(-\Sigma\alpha_1)$$

$$a_2 = a_0(\Sigma\alpha_1\alpha_2)$$

$$a_3 = a_0(-\Sigma\alpha_1\alpha_2\alpha_3)$$

$$\cdots \quad \cdots$$

$$a_n = a_0(-1)^n\alpha_1\alpha_2\cdots\alpha_n$$

Therefore $\Sigma\alpha_1 = -\frac{a_1}{a_0} = -\frac{\text{Coefficient of } x^{n-1}}{\text{Co-efficient of } x^n}$

$$\Sigma\alpha_1\alpha_2 = \frac{a_2}{a_0} = \frac{\text{Co-efficient of } x^{n-2}}{\text{Co-efficient of } x^n}$$

$$\Sigma\alpha_1\alpha_2\alpha_3 = -\frac{a_3}{a_0} = -\frac{\text{Co-efficient of } x^{n-3}}{\text{Co-efficient of } x^n}$$

$$\cdots \cdots$$

$$\alpha_1\alpha_2\cdots\alpha_n = (-1)^n\frac{a_n}{a_0} = (-1)^n\frac{\text{Constant term}}{\text{Co-efficient of } x^n}$$

Clearly, $\Sigma\alpha_1 =$ Sum of the roots

$\Sigma\alpha_1\alpha_2 =$ Sum of the products of the roots taken two at a time.

$\Sigma\alpha_1\alpha_2\cdots\alpha_r =$ Sum of the products of the roots taken r at a time. $(r \leq n)$

Some Particular Cases

(1) Let $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$ be a cubic equation and α, β, γ be its roots.

Then,

$$\Sigma\alpha = \alpha + \beta + \gamma = -\frac{a_1}{a_0}$$

$$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \gamma\alpha = \frac{a_2}{a_0}$$

$$\alpha\beta\gamma = -\frac{a_3}{a_0}$$

(2) Let $\alpha, \beta, \gamma, \delta$ be the roots of the biquadratic equation $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$.

Then $\sum\alpha = \alpha+\beta+\gamma+\delta = -\dfrac{a_1}{a_0}$

$$\sum\alpha\beta = \alpha\beta+\alpha\gamma+\alpha\delta + \beta\gamma+\beta\delta+\gamma\delta = \frac{a_2}{a_0}$$

$$\sum\alpha\beta\gamma = \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta + \beta\gamma\delta = -\frac{a_3}{a_0}$$

$$\alpha\beta\gamma\delta = \frac{a_4}{a_0}$$

Solve: $x^3+6x^2-3x-18=0$, given that the sum of two of the roots is zero.

Soln: Let the roots of the given be α, β, γ

Let $\alpha+\beta=0$ (As it is given that the sum of the two roots is zero.

So, $\beta=-\alpha$

From the relation between roots and co-efficient we have

$$\alpha+\beta+\gamma = -6 \quad \text{—— (i)}$$
$$\alpha\beta+\beta\gamma+\gamma\alpha = -3 \quad \text{—— (ii)}$$
$$\& \quad \alpha\beta\gamma = 18 \quad \text{—— (iii)}$$

Since $\alpha+\beta=0$, So from (i) $\gamma=-6$

From (iii) $\alpha(-\alpha)(-6)=18 \Rightarrow \alpha^2 = 3$

$\Rightarrow \alpha = \pm\sqrt{3}$

Hence the roots are $\sqrt{3}, -\sqrt{3}, -6$.

Homework for the Students :

1. Solve: $x^4+2x^3+5x^2+4x+3=0$, given that product of two ~~of~~ the roots is 1

2. Solve: $6x^3+11x^2-19x+6=0$ having given that two of the roots ~~of its~~ are in ratio 3:4.

~~For YouTube~~
YouTube Link : https://youtu.be/exg03GN3Kgc

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