Study Material for

B.Sc. II (Math (Sub/Gen))

Topic : Differential Equation

Subtopic : D.E. of Ist order and Ist degree

Material Sl. no — 6

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## Linear Equation

General linear D.E. is of the form

$$\frac{dy}{dx} + Py = Q$$

Where P & Q are functions of x or constant.

## Method of Solution

We multiply both sides of D.E by $e^{\int P dx}$.

Then the equation becomes,

$$\frac{dy}{dx} e^{\int P dx} + Py e^{\int P dx} = Q e^{\int P dx}$$

or, $$\frac{d}{dx}\left(y e^{\int P dx}\right) = Q e^{\int P dx}$$

Integrating we get

$$\int d\left(y e^{\int P dx}\right) = \int Q e^{\int P dx} dx$$

or, $$y e^{\int P dx} = \int Q e^{\int P dx} dx + C.$$

Which is the required soln.

Here $e^{\int P dx}$ is called integrating factor of the differential equation.

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## Worked out Examples

Solve.

(1) $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$.

**Soln.** : $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

or, $\frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$

So, I.F. $= e^{\int \frac{1}{1+x^2}dx}$

$= e^{\tan^{-1}x}$

Therefore multiplying both sides of the D.E by I.F. i.e $e^{\tan^{-1}x}$ we get

$$\frac{d}{dx}(y e^{\tan^{-1}x}) = \frac{e^{\tan^{-1}x}\tan^{-1}x}{1+x^2}$$

Integrating we get

$$\int d(y e^{\tan^{-1}x}) = \int \frac{e^{\tan^{-1}x}\tan^{-1}x}{1+x^2}dx$$

or, $y e^{\tan^{-1}x} = \int z e^z dz + C_1$ $\quad$ Let $\tan^{-1}x = z$, So $\frac{dx}{1+x^2} = dz$

$= z\int e^z dz - \int\left(\frac{d}{dz}z \int e^z dz\right)dz$

$= ze^z - \int e^z dz + C_2$

$y e^{\tan^{-1}x} = ze^z - e^z + C$

or, $y e^{\tan^{-1}x} = (\tan^{-1}x - 1)e^{\tan^{-1}x} + C$.

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(2) Solve $\frac{dy}{dx} + \frac{n}{x} y = \frac{a}{x^n}$.

Soln $\frac{dy}{dx} + \frac{n}{x} y = \frac{a}{x^n}$

Which is a linear D.E. of degree one.

I.F. $= e^{\int \frac{n}{x} dx}$

$= e^{n \log x}$

$= e^{\log x^n}$

$= x^n$. $\left[ \text{As } e^{\log P} = P \right]$

Multiplying both sides of the D.E. by $x^n$ we get.

$$x^n \frac{dy}{dx} + n x^{n-1} y = a$$

or, $\frac{d(x^n y)}{dx} = a$

Integrating we get.

$$\int d(x^n y) = \int a\, dx$$

or, $y \cdot x^n = ax + c$.

or, $y = \frac{a}{x^{n-1}} + \frac{c}{x^n}$.

Which is the required general solution.

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## Homework for the Students.

Solve

(i) $x\cos x\frac{dy}{dx} + y(x\sin x + \cos x) = 1$.

(ii) $\frac{dy}{dx} + y\cot x = 2\cos x$

(iii) $\frac{dy}{dx} + \frac{y}{x} = \sin x^2$

(iv) $(1-x^2)\frac{dy}{dx} - xy = 1$

(v) $\frac{dy}{dx} + xy = x^3$

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