

Study Material.

B.Sc. - II (Math Honrs)

Paper - 3

Sequence & Series.

Material Sl. no.  $\rightarrow$  I'I

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# Bounded Sequence

A sequence  $\{x_n\}$  is said to be **bounded above** if there exists a real number  $U$  such that  $x_n \leq U$  for all  $n \in \mathbb{N}$ .

Where  $U$  is an upper bound of the sequence  $\{x_n\}$ .

A sequence  $\{x_n\}$  is said to be **bounded below** if there exists a real number  $L$  such that  $L \leq x_n$ , for all  $n \in \mathbb{N}$ .

Where  $L$  is a lower bound of the sequence  $\{x_n\}$ .

A sequence  $\{x_n\}$  is said to be **bounded** if there exists a real positive number  $M$  such that  $|x_n| \leq M$ , i.e.  $-M \leq x_n \leq M$ .

i.e. a sequence  $\{x_n\}$  is bounded iff it is bounded above as well bounded below.

## Examples

- $\{1, 1/2, 1/3, \dots\}$  is a bounded sequence. As 0 is the greatest lower bound and 1 is the least upper bound.
- $\{1^n, 2^n, 3^n, \dots\}$  is bounded below but unbounded above.



3.  $\{-1, -2, -3, \dots\}$  is bounded above  
but unbounded below.

4.  $\{(-1)^n n\}$  is neither bounded above nor bounded below.

### Monotone Sequence

A sequence  $\{x_n\}$  is said to be **monotone increasing** if  $x_{n+1} \geq x_n \quad \forall n \in \mathbb{N}$ .

A sequence  $\{x_n\}$  is said to be **strictly monotone increasing** if  $x_{n+1} > x_n \quad \forall n \in \mathbb{N}$ .

A sequence  $\{x_n\}$  is said to be **monotone decreasing** if  $x_{n+1} \leq x_n, \quad \forall n \in \mathbb{N}$ .

A sequence  $\{x_n\}$  is said to be **strictly monotone decreasing** if  $x_{n+1} < x_n, \quad \forall n \in \mathbb{N}$ .

### Examples

1.  $\{2, 2^2, 2^3, 2^4, \dots, 2^n, \dots\}$  is strictly monotone increasing sequence.

2. The sequence  $\{1/3^n\}$  is strictly monotone decreasing sequence.

3. The sequence  $\{(-1)^n\}$  is ~~neither~~ <sup>neither</sup> ~~increasing~~ <sup>monotone</sup> ~~decreasing~~ <sup>increasing</sup> ~~neither~~ <sup>neither</sup> ~~increasing~~ <sup>neither</sup> ~~decreasing~~ <sup>neither</sup>.

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