Study Material for

B.Sc. II (Math (Sub/Gen))

Topic: Differential Equation

Sub-topic: D.E. of Ist order and Ist degree

Material Sl. no — 3

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Homogeneous Equation

A function $f(x,y)$ is said to be homogeneous function of degree n if it can be expressed in the form

$$f(x,y) = x^n \phi(y/x)$$

A differential equation of the form $Mdx + Ndy = 0$ is said to be homogeneous if M and N ~~and~~ are homogeneous function ~~of degree~~ of same degree.

How to solve

Let a homogeneous differential equation be

$$Mdx + Ndy = 0 \quad \text{——} \quad (1)$$

Then M can be written as $M = x^n f(y/x)$

N " " $N = x^n g(y/x)$

(1) can be written in the form

$$\frac{dy}{dx} = -\frac{M}{N}$$

$$= -\frac{f(y/x)}{g(y/x)}$$

$$= \phi(y/x) \text{ . say}$$

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We substitute $y/x = v$

i.e $y = vx$

On differentiating we get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Therefore the differential equation takes the form

$$v + x\frac{dv}{dx} = \phi(v)$$

or, $x\frac{dv}{dx} = \phi(v) - v$

or, $\frac{dv}{\phi(v) - v} = \frac{dx}{x}$

Integrating and then substituting the values of v, ~~the~~ solution can be obtained.

Worked out Example

Solve $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$

Soln $x\,dy - y\,dx - \sqrt{x^2+y^2}\,dx = 0$

or, $x\,dy - (y + \sqrt{x^2+y^2})\,dx = 0$

or, $x\,dy = (y + \sqrt{x^2+y^2})\,dx$

or, $\frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$

Let $y = vx$, so, $\frac{dy}{dx} = v + x\frac{dv}{dx}$

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$$v + x\frac{dv}{dx} = \frac{vx + \sqrt{x^2 + v^2x^2}}{x}$$

$$\text{or, } v + x\frac{dv}{dx} = v + \sqrt{1+v^2}$$

$$\text{or, } x\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\text{or, } \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

Integrating we get

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\cancel{\text{or,}}\ \log\left|v + \sqrt{1+v^2}\right| + c_1 = \log x$$

$$\text{or, } \log\left(\frac{y}{x} + \sqrt{1 + y^2/x^2}\right) + \log c = \log x$$

$$\text{or, } c\left(y/x + \frac{\sqrt{x^2+y^2}}{x}\right) = x$$

$$\text{or, } c\left(y + \sqrt{x^2+y^2}\right) = x^2$$

Where c is an integrating constant.

Homework for the students

Solve the following d.E.

(i) $2xy\frac{dy}{dx} = y^2 - x^2$

(ii) $\frac{dy}{dx} = \frac{y}{x} + \tan y/x$

(iii) $x \sin y/x \, dy = (y \sin y/x - x) \, dx$

(iv) $\frac{dy}{dx} + \frac{x^2+3y^2}{y^2+3x^2} = 0$

(v) $y\,dx - x\,dy = \sqrt{x^2-y^2}\,dx$

(vi) $\frac{dy}{dx} = \frac{x-y}{x+y}$.