Study Material

B.Sc. II (Math)

Paper - 4.

Topic: Some important problems of Laplace Transformation

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Find the laplace transformation of the following

1. $(\cosh at - \cos at)$

2. $t^3 e^{-3t}$

$\boxed{\text{Soln.}}$

1. $L(\cosh at - \cos at)$

$= L(\cosh at) - L(\cos at)$ [As L is linear]

$= \dfrac{s}{s^2-a^2} - \dfrac{s}{s^2+a^2}$

$= \dfrac{(s^2+a^2)s - s(s^2-a^2)}{(s^2-a^2)(s^2+a^2)}$

$= \dfrac{s^3+a^2s - s^3+a^2s}{s^4-a^4}$

$= \dfrac{2a^2s}{(s^4-a^4)}$

2. $L(t^3 e^{-3t})$

We know that $L(t^3) = \frac{3!}{s^{3+1}} = \frac{6}{s^4} = F(s)$ (say)

Using first shifting theorem we get

$$L(t^3 e^{-3t}) = \cancel{F(s+a)}$$
$$= F(s-(-3))$$
$$= F(s+3)$$
$$= \frac{6}{(s+3)^4}$$

Apply Laplace transformation to solve the following differential equation.

1. $\frac{d^2y}{dt^2} + t\frac{dy}{dt} = y$, $y(0) = 0$, $\frac{dy}{dt} = 1$, at $t = 0$

$\boxed{\text{Soln.}}$ Given differential equation can be written as:

$$y'' + ty' - y = 0.$$

Applying Laplace transformation on the both sides of the above equation we get

$$L(y'') + L(ty') - L(y) = 0$$

or, $s^2L(y) - sy(0) - y'(0) + (-1)^1 \frac{d}{ds} L(y') - L(y) = 0$

or, $s^2L(y) - s.0 - 1 - \frac{d}{ds}[sL(y) - y(0)] - L(y) = 0$

or, $(s^2-1)L(y) - 1 - \frac{d}{ds}[sL(y)] = 0$

or, $(s^2-1)L(y) - 1 - L(y) - s\frac{d}{ds}L(y) = 0$

or, Let $L(y) = x$.

Then above equation become.

$$(s^2-2)x - 1 - s\frac{dx}{ds} = 0$$

or, $s\frac{dx}{ds} = (s^2-2)x - 1$

or, $\frac{dx}{ds} = (s - 2/s)x - 1/s$

or $\frac{dx}{ds} + (2/s - s)x = -1/s$

Which is linear in x.

So I.F. $e^{\int(2/s - s)\,ds}$

$= e^{2\log s - s^2/2}$

$= e^{\log s^2} \cdot e^{-s^2/2}$

$= s^2 e^{-s^2/2}$

Multiplying the above equation by I.F. we get

$$d(x s^2 e^{-s^2/2}) = -s e^{-s^2/2} ds$$

Integrating we get

$x s^2 e^{-s^2/2} = -\int s e^{-s^2/2} dx$ Let $-s^2/2 = z$

or, $-s\,ds = dz$

$= \int e^z dz$

$x s^2 e^{-s^2/2} = e^z + c$

or, $x \cdot s^2 e^{-s^2/2} = e^{-s^2/2} + c$

or, $x s^2 = 1 + c e^{s^2/2}$

or, $L(y) \cdot s^2 = 1 + c e^{s^2/2}$

or, $L(y) = 1/s^2 + c\frac{e^{s^2/2}}{s^2}$

or, $y = L^{-1}\left(\frac{1}{s^2}\right) + L^{-1}\left(\frac{c}{s^2} e^{-s^2/2}\right)$

or, $y = t + c\, L^{-1}\left(\frac{1}{s^2} e^{-s^2/2}\right)$

When $t=0$, $y=0$. So, $c=0$

Therefore $y = t$. is the required solution,

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