Study Material

B.Sc. - II (Math)

Paper - A.

Topic : Some important Problems of Laplace Transformation

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Some Important Problems of Laplace Transformation

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1. Find the Laplace transformation of

$$e^{-3t}(2\cos 5t - 3\sin 5t)$$

Soln We have to find

$$L\{e^{-3t}(2\cos 5t - 3\sin 5t)\}$$

Now we have $L(2\cos 5t - 3\sin 5t)$

$$= 2L(\cos 5t) - 3L(\sin 5t)$$

$$= 2\cdot\frac{s}{s^2+5^2} - 3\cdot\frac{5}{s^2+3^2}$$

$$= \frac{2s-15}{s^2+5^2}$$

Hence by First shifting theorem

$$L(e^{-3t}(2\cos 5t - 3\sin 5t))$$

$$= \frac{2(s+3)-15}{(s+3)^2+5^2}$$

$$= \frac{2s-9}{s^2+6s+34}$$

Find the inverse Laplace transformation of the following

1. $\frac{s+2}{s^2-4s+13}$, ② $\frac{4s+5}{(s-1)^2(s+2)}$

Soln ① $L^{-1}\left(\frac{s+2}{s^2-4s+13}\right)$

$= L^{-1}\left(\frac{s+2}{(s-2)^2+3^2}\right)$

$= e^{2t} L^{-1}\left(\frac{s+4}{s^2+3^2}\right)$

$= e^{2t}\left[L^{-1}\left(\frac{s}{s^2+3^2}\right) + 4L^{-1}\left(\frac{1}{s^2+3^2}\right)\right]$

$= e^{2t}\left[\cos 3t + \frac{4}{3}\sin 3t\right]$

② $L^{-1}\left(\frac{4s+5}{(s-1)^2(s+2)}\right)$

Let $\frac{4s+5}{(s-1)^2(s+2)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+2}$

$= \frac{(s+2)(s-1)A + (s+2)B + (s-1)^2 C}{(s-1)^2(s+2)}$

or, $4s+5 = (s+2)(s-1)A + (s+2)B + (s-1)^2 C$

Putting $s=1$, we get $3B = 9$ or, $B=3$

" $s=-2$ we get $9C = -3$ or, $C = -1/3$

" $s=0$, we get $-2A + 2B + C = 5$

or, $-2A = 5 - 6 + 1/3 = \frac{-3+1}{3}$

So $A = 1/3$

So $L^{-1}\left(\frac{4s+5}{(s-1)^2(s+2)}\right)$

$= L^{-1}\left(\frac{1/3}{s-1}\right) + L^{-1}\left(\frac{3}{(s-1)^2}\right) + L^{-1}\left(\frac{-1/3}{s+2}\right)$

$= \frac{1}{3}L^{-1}\left(\frac{1}{s-1}\right) + 3L^{-1}\left(\frac{1}{(s-1)^2}\right) - \frac{1}{3}L^{-1}\left(\frac{1}{s+2}\right)$

$= \frac{1}{3}e^{t}L^{-1}\left(\frac{1}{s}\right) + 3e^{2t}L^{-1}\left(\frac{1}{s^2}\right) - \frac{1}{3}e^{-2t}L^{-1}\left(\frac{1}{s}\right)$

$= \frac{1}{3}e^{t} + 3e^{2t}\cdot\frac{t^{2-1}}{\lfloor 2-1} - \frac{1}{3}e^{-2t}$

$= \frac{1}{3}e^{t} + 3te^{2t} - \frac{1}{3}e^{-2t}$