Study Material

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B.Sc.-II (Math)

Paper – 4.

Topic : Some important problems of Laplace Transformation

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Some Important Problems of Laplace Transformation

Find the Laplace Transformation of the following

1. $t^3 e^{-3t}$, 2. $t \sin at$.

$\boxed{\text{Soln}}$ 1. $L(t^3 e^{-3t})$

We know that $L(t^3) = \dfrac{3!}{s^{3+1}} = \dfrac{6}{s^4}$

From first shifting theorem we have,

$$L(t^3 e^{-3t}) = \frac{6}{(s+3)^4}$$

2. We have to find $L(t \sin at)$

we know that $L(\sin at) = \dfrac{a}{s^2 + a^2}$.

So $L(t \sin at) = (-1)^1 \frac{d}{ds} L(\sin at)$

$$= -\frac{d}{ds}\left(\frac{a}{s^2+a^2}\right)$$

$$= +\frac{a \cdot 2s}{(s^2+a^2)^2}$$

$$= \frac{2as}{(s^2+a^2)^2}$$

Find the inverse laplace transformation of the following

1. $\frac{s^2-3s+4}{s^3}$, 2. $\frac{5s+3}{(s-1)(s^2+2s+5)}$

$\boxed{\text{Soln}}$ 1. We have to find $L^{-1}\left(\frac{s^2-3s+4}{s^3}\right)$

$$= L^{-1}\left(\frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^3}\right)$$

$$= L^{-1}\left(\frac{1}{s}\right) - 3L^{-1}\left(\frac{1}{s^2}\right) + 4L^{-1}\left(\frac{1}{s^3}\right)$$

$$= 1 - 3 \cdot \frac{t}{1!} + 4 \cdot \frac{t^2}{2!}$$

$$= 1 - 3t + 2t^2$$

2. $L^{-1}\left(\frac{5s+3}{(s-1)(s^2+2s+5)}\right)$

$$= L^{-1}\left(\frac{5(s+1)-2}{(s+1-2)((s+1)^2+2^2)}\right)$$

$$= e^{-t} L^{-1}\left(\frac{5s-2}{(s-2)(s^2+2^2)}\right) \quad \text{--- (1)}$$

Let $\frac{5s-2}{(s-2)(s^2+2^2)} = \frac{A}{s-2} + \frac{Bs+C}{s^2+4}$

$$= \frac{(s^2+4)A + (s-2)(Bs+C)}{(s-2)(s^2+4)}$$

or, $5s-2 = (s^2+4)A + (s-2)(Bs+C)$

Putting $s=2$, we get $8 = 8A \Rightarrow A = 1$

Putting $s=0$, we get

$$-2 = 4 - 2\cdot C$$

$$\text{or, } -2C = -6$$

$$\text{or, } C = 3$$

Putting $s=1$, we get $3 = 5A - (B+C)$

$$\text{or, } -2 = -(B+C)$$

$$\text{or, } B+C = 2$$

$$\text{or, } B = -1$$

Therefore the equation (i) can be written as

$$L^{-1}\left(\frac{5s+3}{(s-1)(s^2+2s+5)}\right) = e^{-t}L^{-1}\left(\frac{1}{s-1} + \frac{-s+3}{s^2+2^2}\right)$$

$$= e^{-t}\left[L^{-1}\left(\frac{1}{s-1}\right) + L^{-1}\left(\frac{3}{s^2+2^2}\right) - L^{-1}\left(\frac{s}{s^2+2^2}\right)\right]$$

$$= e^{-t}\left[e^{t}L^{-1}\left(\frac{1}{s}\right) + \frac{3}{2}L^{-1}\left(\frac{2}{s^2+2^2}\right) - \cos 2t\right]$$

$$= e^{-t}\left[e^{t} + \frac{3}{2}\sin 2t - \cos 2t\right]$$

$$= 1 + \frac{e^{-t}}{2}(3\sin 2t - 2\cos 2t)$$

Q. Find the Laplace transformation of $\sin^3 2t$.

$\boxed{\text{Soln:}}$ $L(\sin^3 2t)$

$$= \frac{1}{4}L(4\sin^3 2t)$$

$$= \frac{1}{4}L(3\sin 2t - \sin 6t)$$

$$= \frac{3}{4}L(\sin 2t) - \frac{1}{4}L(\sin 6t)$$

$$= \frac{3}{4} \cdot \frac{2}{s^2+2^2} - \frac{1}{4} \cdot \frac{6}{s^2+6^2}$$

$$= \frac{6(s^2+6^2 - s^2 - 2^2)}{4(s^2+2^2)(s^2+6^2)}$$

$$= \frac{6 \cdot 32^8}{4 (s^2+2^2)(s^2+6^2)}$$

$$= \frac{48}{(s^2+2^2)(s^2+6^2)}$$

Find the Inverse Laplace transformation of $\frac{4s+5}{(s-1)^2(s+2)}$

$\boxed{\text{Soln}}$ $L^{-1}\left(\frac{4s+5}{(s-1)^2(s+2)}\right)$

Let $\frac{4s+5}{(s-1)^2(s+2)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+2}$

$$= \frac{(s-1)(s+2)A + (s+2)B + (s-1)^2 C}{(s-1)^2(s+2)}$$

or, $4s+5 = (s-1)(s+2)A + (s+2)B + (s-1)^2 C$

Putting $s = 1$, we get $9 = 3B$ or $B = 3$

Putting $s = -2$, we get $-3 = 9C$ or, $C = -\frac{1}{3}$

Putting $s = 0$, we get $5 = -2A + 2B + C$

or, $2A = 6 - \frac{1}{3} - 5$

or, $2A = 1 - \frac{1}{3} = \frac{2}{3}$

or, $A = \frac{1}{3}$.

or, $L^{-1}\left(\frac{4s+5}{(s-1)^2(s+2)}\right) = L^{-1}\left(\frac{\frac{1}{3}}{s-1}\right) + L^{-1}\left(\frac{1}{(s-1)^2}\right) - \frac{1}{3} L^{-1}\left(\frac{1}{s+2}\right)$

$= \frac{1}{3} e^t L^{-1}(\frac{1}{s}) + e^t L^{-1}\left(\frac{1}{s^2}\right) - \frac{1}{3} e^{-2t} L^{-1}\left(\frac{1}{s}\right)$

$= \frac{1}{3} \cdot e^t + e^t \cdot t - \frac{1}{3} e^{-2t}$

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