Study Material

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B.Sc. II (Math)

Paper – 4.

Topic: Some Important Problems of Laplace Transformation.

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Apply Laplace Transformation to solve the following differential equation.

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = e^{-t}, \quad y(0) = 0, \quad y'(0) = 1$$

Soln. Given D.E. can be written as

$$y'' + 4y' + 3y = e^{-t}.$$

Applying Laplace Transformation on the both sides of the above equation we get

$$L(y'') + 4L(y') + 3L(y) = L(e^{-t})$$

$$\text{or, } s^2L(y) - sy(0) - y'(0) + 4sL(y) - 4y(0) + 3L(y) = L(e^{-t})$$

$$\text{or, } s^2L(y) - 1 + 4sL(y) + 3L(y) = \frac{1}{s+1}$$

$$\text{or, } (s^2+4s+3)L(y) = 1+\frac{1}{s+1}$$

$$= \frac{s+2}{s+1}$$

$$\text{or, } L(y) = \frac{s+2}{(s+1)(s^2+4s+3)}$$

$$\text{or, } y = L^{-1}\left(\frac{s+2}{(s+1)(s^2+3s+s+3)}\right)$$

$$= L^{-1}\left(\frac{s+2}{(s+1)(s+3)(s+1)}\right)$$

$$= L^{-1}\left(\frac{s+2}{(s+1)^2(s+3)}\right) \quad \text{---(1)}$$

$$\text{Let } \frac{s+2}{(s+1)^2(s+3)} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{s+3}$$

$$= \frac{(s+1)(s+3)A + (s+3)B + (s+1)^2C}{(s+1)^2(s+3)}$$

$$\text{or, } s+2 = (s+1)(s+3)A + (s+3)B + (s+1)^2C$$

Putting $s=-1$, we get

$$1 = 2B \quad \text{or, } B = 1/2$$

Again putting $s = -3$ we get

$-1 = 4C$

or, $C = -1/4$.

Lastly putting $s = 0$, we get

$2 = 3A + 3B + C$

or, $2 = 3A + 3/2 - 1/4$

or, $3A = 2 - 3/2 + \frac{1}{4}$

or, $= \frac{8-6+1}{4}$

$= 3/4$

or, $A = 1/4$

Therefore from (1) we have

$$y = L^{-1}\left(\frac{\frac{1}{4}}{s+1}\right) + L^{-1}\left(\frac{\frac{1}{2}}{(s+1)^2}\right) + L^{-1}\left(\frac{-1/4}{s+3}\right)$$

$$= \frac{1}{4}L^{-1}\left(\frac{1}{s+1}\right) + \frac{1}{2}L^{-1}\left(\frac{1}{(s+1)^2}\right) - \frac{1}{4}L^{-1}\left(\frac{1}{s+3}\right)$$

$$= \frac{1}{4}e^{-t}L^{-1}\left(\frac{1}{s}\right) + \frac{1}{2}e^{-t}L^{-1}\left(\frac{1}{s^2}\right) - \frac{1}{4}e^{-3t}L^{-1}\left(\frac{1}{s}\right)$$

$$\text{or, } y = \frac{1}{4}e^{-t} + \frac{1}{2}e^{-t}\cdot t - \frac{1}{4}e^{-3t}$$