Study Material

B.Sc. I (Math (Hons))

Topic: Matrices

Material Sl. no. 5

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Matrices

Hermitian Matrix

A square matrix A is said to be ~~hard~~ hermitian if $A = (\overline{A})^t$

$(\overline{A})^t$ is called conjugate transpose of A.

i.e if $A = [a_{ij}]_{m \times n}$

Then ~~o~~ if A is hermitian then $a_{ij} = \overline{a_{ji}}$ $\forall\, i, j$.

Example

$A = \begin{pmatrix} 2 & 1-2i \\ 1+2i & 1 \end{pmatrix}$ is a hermitian matrix.

As $\overline{A} = \begin{pmatrix} 2 & 1+2i \\ 1-2i & 1 \end{pmatrix}$

$$(\bar{A})^t = \begin{bmatrix} 2 & 1-2i \\ 1+2i & -1 \end{bmatrix} = A.$$

Q. If A is Hermitian matrix, then all of its diagonal element is real.

Soln: Let $A = [a_{ij}]_{n\times n}$ be Hermitian.

Then $a_{ij} = \overline{a_{ji}}$ $\forall$ i, j

Therefore $a_{ii} = \overline{a_{ii}}$

Let $a_{ii} = x + iy$.

Then $a_{ii} = \overline{a_{ii}}$ gives.

$$x + iy = x - iy$$

$$\text{or, } 2iy = 0$$

$$\text{or, } y = 0$$

Hence every diagonal element of a Hermitian matrix is ~~purely~~ real.

Skew Hermitian Matrix

A square matrix A is said to be skew-Hermitian if $A = -(\bar{A})^t$

i.e if $A = [a_{ij}]_{n\times n}$ is ~~skew~~ skew Hermitian then $a_{ij} = -\overline{a_{ji}}, \forall i, j$.

$\boxed{\text{Example.}}$

$$A = \begin{pmatrix} 0 & 2+3i \\ -2+3i & -i \end{pmatrix}$$ is skew Hermitian

As $$\overline{A} = \begin{pmatrix} 0 & 2-3i \\ -2-3i & i \end{pmatrix}$$

$$\overline{A}^t = \begin{pmatrix} 0 & -2-3i \\ 2-3i & i \end{pmatrix}$$

$$= -\begin{pmatrix} 0 & 2+3i \\ -2+3i & -i \end{pmatrix}$$

$$= -A$$

Q. If A is skew-Hermitian then the diagonal element must purely imaginary or zero.

Soln: Let $A = [a_{ij}]_{n\times n}$ is skew Hermitian

Then $a_{ij} = -\overline{a_{ji}} \quad \forall i, j$

Therefore $a_{ii} = -\overline{a_{ii}}$

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or, $a_{ii} + \overline{a_{ii}} = 0$

So a_{ii} must be either zero or purely imaginary.

Q. ~~If~~ A & B are Hermitian then AB+BA is Hermitian.

Soln: A & B are Hermitian

So $(\bar{A})^t = A$ & $(\bar{B})^t = B$

Now $\left[\overline{(AB+BA)}\right]^t = [AB+BA]^\theta$ $\left[\text{denoting } (\bar{A})^t = A^\theta\right]$

$$= (AB)^\theta + (BA)^\theta$$

$$= B^\theta A^\theta + A^\theta B^\theta$$

$$= A^\theta B^\theta + B^\theta A^\theta$$

$$= AB + BA$$

So AB+BA is Hermitian.

- AB − BA is skew Hermitian if A & B are Hermitian.

Proof is similar as above.