Study Material

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B.Sc. I (Math (Hons))

Topic: Matrices

Material Sl. no. 6

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Matrices

Unitary Matrix

A square matrix is said to be unitary matrix if $A^{\theta}A = I$, where $A^{\theta} = (\bar{A})^t$

Example

$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is a unitary matrix.

Since $\bar{A} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$$(\bar{A})^t = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$A^{\theta}A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I_2 .$$

Q. If A is unitary then $|\det A| = 1$

Soln: Let A is unitary.

Then $A^\theta A = I$

or, $\det(A^\theta A) = \det I$

or, $\det A^\theta \det A = 1$

or, $\det \overline{A} \; \det A = 1$

~~Or, $|\det A|^2 = 1$~~

~~$\det(A\overline{A}) = 1$~~

~~or, det~~

or, $\overline{\det A} \cdot \det A = 1$

or, $|\det A| = 1$

This completes the proof.

Note: If A is ~~orth~~ unitary then

$A A^\theta = I$

$$\boxed{\text{Adjoint of a Square Matrix}}$$

Let $A = [a_{ij}]$ be a $n \times n$ matrix.

Then ~~a matrix~~ $B = [A_{ij}]$ is a matrix ~~matrix~~ whose elements are the co-factors

of the corresponding elements of A in det A.
i.e A_{ij} is the co-factor of a_{ij} in det(A)

Transpose of the matrix B is called the adjoint of A and it denoted by adj A

Important Result

$$(\text{adj}\, A)\, A = A(\text{adj}\, A) = |A|\, I_n.$$

Inverse of matrix

Let A be a square matrix of order n.
If there exists a matrix B such that
$AB = BA = I_n$, then the matrix B is said to be the inverse of A.

Q. Inverse of a invertable matrix is unique.

Proof: Let A be a invertible matrix of order n.
If possible let it has two inverse B & C.
Then $AB = BA = I_n$ & $AC = CA = I_n$.

Now $B = BI_n$
$= B(AC)$
$= (BA)C = I_n C = C$

Hence the inverse is unique.