Study Material

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B.Sc. I (Math (Hons))

Topic: Matrices

Paper:

Material Sl. no.:

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Matrices

Existence of Inverse

The necessary and sufficient condition for a square matrix A is invertible is $|A| \neq 0$.

Proof

Necessary Part:

Let A be a $n \times n$ square matrix and a matrix B be its inverse.

Then $AB = BA = I$.

Therefore $d(AB) = \det I$.

or, $\det A \det B = 1$.

This shows that $\det A \neq 0$.

Sufficient Part:

Let A be a $n \times n$ square matrix such that $\det A \neq 0$.

Let B be a matrix, s.t. $B = \dfrac{\operatorname{adj} A}{\det A}$.

This matrix exists as $\det A \neq 0$

Now
$$AB = A\frac{(\text{adj} A)}{\det A}$$
$$= \frac{A\,\text{adj} A}{\det|A|} = \frac{\det(A)\cdot I_n}{\det A}$$
$$= I_n$$

Again
$$BA = \left(\frac{\text{adj} A}{\det A}\right) \cdot A$$
$$= \frac{(\text{adj} A)\, A}{\det A}$$
$$= \frac{(\det A)\, I_n}{\det A} \quad [\because (\text{adj} A)\, A = |A| I_n]$$
$$= I_n$$

Therefore $AB = BA = I$.
Consequently A is invertible and $A^{-1} = B$.

$\boxed{\text{Non singular matrix}}$

A square matrix is said to be non-singular if $|A| \neq 0$

$\boxed{\text{Singular matrix}}$

A square matrix A is said to be singular if $|A| = 0$.

* Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

Where $|A| \neq 0$, $|B| \neq 0$, $|AB| \neq 0$

Pf:

Let $D = B^{-1}A^{-1}$

Now $(AB)D = (AB)(B^{-1}A^{-1})$

$= A(BB^{-1})A^{-1}$

$= A(IA^{-1})$

$= AA^{-1}$

$= I$.

$D(AB) = (B^{-1}A^{-1})(AB)$

$= B^{-1}(A^{-1}A)B$

$= B^{-1}(IB)$

$= B^{-1}B$

$= I$.

This shows that D is the inverse of AB

i.e $B^{-1}A^{-1}$ is the ~~is~~ the inverse of AB.

$\therefore (AB)^{-1} = B^{-1}A^{-1}$.

Problem Find the inverse of the matrix.

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

Soln:

$$|A| = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

So A^{-1} exists.

$$\operatorname{adj} A = \text{transpose of} \begin{bmatrix} \cos x & +\sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\operatorname{adj} A}{|A|} = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$