Study Material

B.Sc. I (Math (Hons))

Topic: Matrices

Paper: 1

Material Sl. no. - 8

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Matrices.

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Submatrix

Let A be a $m \times n$ matrix. Then any matrix obtained from A by removing some rows and some columns is called a submatrix of A.

Example

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 0 & 1 & 2 \end{bmatrix}$

Then $B = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \\ 9 & 0 & 1 \end{bmatrix}$ is a submatrix of A obtained from A by removing 4th row.

Rank of matrix

Let A be a $m \times n$ matrix. A number r is said to be the rank of the matrix A if there exists atleast one submatrix of A of order r, whose determinant is non zero and every submatrix of order $(r+1)$ (if it exists) having ~~determinant~~ determinant value zero.

Note:

1. If A be a square matrix of order n, and if $|A| \neq 0$.
Then the rank of $A = n$.

2. If A be a singular matrix of order n.
Then rank of A must be $\leq n-1$.

* Rank of matrix A is denoted by rank (A) or $r(A)$

Example.

Find the rank of the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

Soln/ Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{vmatrix}$

$= 1(-9+8) - 1(6-12) - 1(-4+9)$

$= -1 + 6 - 5$

$= 0$

Now $\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix}$

$= -3 - 2$

$= -5 \neq 0$

So the rank of the matrix A is 2.

2. Let $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rank of $I_3 = 3$ as $\det |I_3| = 1$

Properties.

1. rank of A = rank A^t.

Proof: Let A be a matrix such that rank of $A = r$.

Then there exists atleast a submatrix of order r whose determinant is non zero.

Let one such submatrix be B.

Let B^t be the transpose of B.

Then $|B^t| = |B| \neq 0$

Again B^t is a submatrix A^t.

Since B^t is a square submatrix of A^t and $|B^t| = 0$

So rank of $A^t \geq r$.

If possible let rank of $A^t = r_1$; where $r_1 > r$.

Then A^t contains a square submatrix of of order r, whose determinant is non zero

~~Consequently~~
Let it be C.

Then C^t is a submatrix of A^t of order r, whose determinant is non zero.

~~So~~ Therefore the rank of A is $r_1 > r$

Which is a contradiction.

Therefore our supposition is wrong and rank of $A^t = r$.

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