Study Material

B.Sc. I (Math (Hons))

Topic: Matrices

Paper: I

Material Sl. no.: - 9

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Matrices

Elementary Operations

1. Interchange of any two rows or columns.
2. Multiplication of any row or column by a non zero number.
3. Addition ~~of~~ to elements of one row or column by the corresponding elements of another row ~~not~~ multiplied by a non zero number.

If this ~~ab~~ above mentioned elementary operation applied to a row, then the operation is called elementary row operation and if this applied to column then it is said to be elementary column operation.

Notation

- If we interchang ith and jth row (or column) of a matrix, then this elementary row (column) operation is denoted by $R_i \leftrightarrow R_j$ ($C_i \leftrightarrow C_j$)
- If we multiply ith row (column) by a non zero constant m, then this elementary row (column) operation is denoted by $R_i \to mR_i$. ($C_i \to mC_i$)
- If we add to ith row (column) by K-times of jth row, then this elementary row (column) operation is denoted by

$$R_i' = R_i + KR_j \quad (C_i' = C_i + KC_j)$$

Important note:

Rank of a matrix remains unchanged under elementary operation.

Normal Form

If A be a $m \times n$ matrix and if order of A = r, then A can be reduced to the form $\begin{bmatrix} I_r & O \\ O & O \end{bmatrix}$.

Problem

Reduce the matrix $\begin{bmatrix} 9 & 7 & 3 & 6 \\ 5 & -1 & 4 & 1 \\ 6 & 8 & 2 & 4 \end{bmatrix}$ to normal form and find its rank.

Soln:
$$\begin{bmatrix} 9 & 7 & 3 & 6 \\ 5 & -1 & 4 & 1 \\ 6 & 8 & 2 & 4 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 5 & -1 & 4 & 1 \\ 9 & 7 & 3 & 6 \\ 6 & 8 & 2 & 4 \end{bmatrix}$$

$$\xrightarrow{C_1 \leftrightarrow C_4} \begin{bmatrix} 1 & -1 & 4 & 5 \\ 6 & 7 & 3 & 9 \\ 4 & 8 & 2 & 6 \end{bmatrix} \xrightarrow[\substack{C_3' = C_3 - 4C_1 \\ C_4' = C_4 - 5C_1}]{C_2' = C_2 + C_1}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 13 & -21 & -21 \\ 4 & 12 & -14 & -14 \end{bmatrix} \xrightarrow{C_4' = C_4 - C_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 6 & 13 & -21 & 0 \\ 4 & 12 & -14 & 0 \end{bmatrix}$$

$$\xrightarrow[R_3' = R_3 - 4R_1]{R_2' = R_2 - 6R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 13 & -21 & 0 \\ 0 & 12 & -14 & 0 \end{bmatrix} \xrightarrow{R_2' = R_2 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -7 & 0 \\ 0 & 12 & -14 & 0 \end{bmatrix}$$

$$\xrightarrow{C_3' = C_3 + 7C_2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 12 & 70 & 0 \end{bmatrix} \xrightarrow{R_3' = R_3 - 12R_2}$$

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$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 70 & 0 \end{bmatrix} \xrightarrow{\frac{1}{70}R_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

~~Which~~ So the normal form is $[I_3, 0]$

Therefore the rank of the matrix is 3