

Study Material

B.Sc. II (Math)

Paper - 1

Topic - Matrices

Prepared by: Rajat Subhramdas

Assistant Professor

Dr. L.K.V.D. College, Tajpur

Email: rajatdas10@gmail.com

Matrices

Q. Show that the equations are consistent and solve them

$$5x + 3y + 7z = 4$$

$$3x + 26y + 2z = 9$$

$$7x + 2y + 10z = 5$$

Soln. The given system of equations can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

or, $AX = B$

The augmented matrix

$$[A \ B] = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array} \right]$$

$\frac{1}{5} R_1 \rightarrow$

$$\left[\begin{array}{ccc|c} 1 & 3/5 & 7/5 & 4/5 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array} \right]$$

$$\text{or, } \begin{array}{l} R_2 - 3R_1 \\ R_3 - 7R_1 \end{array} \rightarrow \begin{bmatrix} 1 & 3/5 & 7/5 & 1/5 \\ 0 & 11/5 & -1/5 & 3/5 \\ 0 & -1/5 & 1/5 & -3/5 \end{bmatrix}$$

$$\frac{1}{11}R_2 \rightarrow \begin{bmatrix} 1 & 3/5 & 7/5 & 1/5 \\ 0 & 1/5 & -1/5 & 3/5 \\ 0 & -1/5 & 1/5 & -3/5 \end{bmatrix}$$

$$R_3 + R_2 \rightarrow \begin{bmatrix} 1 & 3/5 & 7/5 & 1/5 \\ 0 & 1/5 & -1/5 & 3/5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus we have $\text{rank } A = \text{rank } [A \ B]$
 Therefore the given system of equation is consistent.

Therefore the given system is equivalent to

$$x + 3/5 y + 7/5 z = 1/5$$

$$1/5 y - 1/5 z = 3/5$$

Let $z = k$.

Then $1/5 y = 3/5 - 1/5 k$

or, $11y = 3 - k$

or, $y = \frac{3 - k}{11}$

Again $5x = 4 - 3y - 7z$

$= 4 - 3 \left(\frac{3 - k}{11} \right) - 7k$

$= \frac{44 - 9 + 3k + 77k}{11}$

$$5x = \frac{80k + 35}{11}$$

$$\text{or, } x = \frac{16k + 7}{11}$$

Q. Find the rank of the matrix A where

$$A = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{vmatrix}$$

Soln:

$$|A| = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{vmatrix}$$

$$= -2(0+12) - 2(0+6) - 3(-4+1)$$

$$= -24 - 12 + 9$$

$$= -36 + 9$$

$$= -27 \neq 0$$

∴ rank of A = 3

Q. Find the rank of the matrix

$$A = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{vmatrix}$$

$$\text{Soln: } |A| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{vmatrix}$$

$$= (-9+8) - 1(6-12) - 1(-4+9)$$

$$= -1 + 6 - 5$$

$$= 0$$

So rank $A \neq 3$.

Now $B = \begin{bmatrix} 1 & 1 \\ 2 & -3 \end{bmatrix}$ is a submatrix of A .

$$|B| = -3 - 2$$

$$= -5$$

$$\neq 0$$

Therefore rank of $A = 2$.