B.Sc. II (Math (Hons))

Study Material

Paper – 4

Topic : Laplace Transformation

Material Sl. no – 2

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Laplace Transformation

Th: Laplace transformation is linear,
i.e if $f(t)$ and $g(t)$ be any two functions
Then $L(af(t)+bg(t)) = aL(f(t)) + bL(g(t))$

for any constant a & b.

Proof: Let $f(t)$ and $g(t)$ be two functions whose laplace transformation exists.
Let a & b be any two constant.

Now. $L(af(t) + bg(t))$

$$= \int_0^\infty e^{-st}\left[af(t) + bg(t)\right] dt$$

$$= \int_0^\infty e^{-st} af(t)\, dt + \int_0^\infty e^{-st} bg(t)\, dt$$

$$= a\int_0^\infty e^{-st} f(t)\, dt + b\int_0^\infty e^{-st} g(t)\, dt$$

$$= aL(f(t)) + bL(g(t))$$

This shows that ~~the~~ laplace transformation is linear.

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Find the Laplace transformation of the following function.

① $\frac{1}{2}\left(e^{at} + e^{-at}\right)$

Soln:
$$L\left(\frac{1}{2}\left(e^{at} + e^{-at}\right)\right)$$

$$= \frac{1}{2}L(e^{at}) + \frac{1}{2}L(e^{-at}) \quad [\text{Using linearity prop.}]$$

$$= \frac{1}{2}\cdot\frac{1}{s-a} + \frac{1}{2}\cdot\frac{1}{s+a}$$

$$= \frac{1}{2}\cdot\frac{s+a+s-a}{s^2-a^2}$$

$$= \frac{1}{2}\cdot\frac{2s}{s^2-a^2}$$

$$= \frac{s}{s^2-a^2}, \quad s>a \ \& \ s>-a$$

$$= \frac{s}{s^2-a^2}, \quad a<s, \ a>-s$$

$$= \frac{s}{s^2-a^2}, \quad -s<a<s$$

$$= \frac{s}{s^2-a^2}, \quad |a|<s$$

$$= \frac{s}{s^2-a^2}, \quad s>|a|$$

Note: $L(\cosh at) = \frac{s}{s^2-a^2}, \quad s > |a|$

2. find the laplace transformation of ~~$\cosh at$~~ $\sinh at$.

Soln: we know that $\sinh(at) = \frac{e^{at}-e^{-at}}{2}$

$\therefore L(\sinh at) = L\left(\frac{1}{2}e^{at} - \frac{1}{2}e^{-at}\right)$

$= \frac{1}{2}L(e^{at}) - \frac{1}{2}L(e^{-at})$ (by linearity of L)

$= \frac{1}{2}\frac{1}{s-a} - \frac{1}{2}\frac{1}{s+a}, \quad s > a, s > -a$

$= \frac{1}{2}\frac{s+a-s+a}{(s-a)(s+a)}, \quad s > a, -s < a$

$= \frac{a}{s^2-a^2}, \quad -s < a, \; a < s$

$= \frac{a}{s^2-a^2}, \quad -s < a < s$

$= \frac{a}{s^2-a^2}, \quad |a| < s.$

So $L(\sinh at) = \frac{a}{s^2-a^2}, \quad |a| < s.$

③ Find the laplace transformation of

$$f(t) = |t-1| + |t+1| , \quad t \geq 0$$

Soln $f(t) = |t-1| + |t+1|, \quad t \leq 0$

Therefore $f(t) = (1-t) + (t+1), \quad 0 \leq t \leq 1$

$$= 2 \quad , \quad 0 \leq t \leq 1$$

& $f(t) = t-1+t+1, \quad t \geq 1$

$$= 2t, \quad t \geq 1$$

Hence $L(f(t)) = \int_0^{\infty} e^{-st} f(t)\, dt$.

$$= \int_0^1 e^{-st}\, 2\, dt + \int_1^{\infty} e^{-st} \cdot 2t\, dt$$

$$= 2\int_0^1 e^{-st} dt + 2\int_1^{\infty} e^{-st} t\, dt$$

$$= 2\left[\frac{e^{-st}}{-s}\right]_0^1 + 2\left[t\frac{e^{-st}}{-s}\right]_1^{\infty} - 2\left[\int_1^{\infty} \frac{e^{-st}}{-s}\, dt\right]$$

$$= \frac{2}{s}\left[1 - e^{-s}\right] + \frac{2}{s}\left[e^{-s} - 0\right] + \frac{2}{s}\left[\frac{e^{-st}}{-s}\right]_1^{\infty}$$

$$= \frac{2}{s} + \frac{2}{s^2}\left[e^{-s} - 0\right]$$

$$= \frac{2}{s}\left[1 + \frac{e^{-s}}{s}\right]$$