B.Sc. II (Math (Hons))

Study Material

Paper – 4

Topic : Laplace Transformation

Material Sl. no – 4

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Laplace Transformation

First shifting theorem

If $L(f(t)) = F(s)$, for $s > K$

Then $L(e^{at} f(t)) = F(s-a)$, for $s > a+K$

Proof

~~We know that,~~

Let $L(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)\,dt$; holds for $s > K$

Now $L(e^{at} f(t)) = \int_0^\infty e^{-st} \cdot e^{at} f(t)\,dt$

$$= \int_0^\infty e^{-(s-a)t} f(t)\,dt$$

$= F(s-a)$, holds when $s-a > K$

$= F(s-a)$, holds for $s > a+K$

Second shifting Theorem

If $L(f(t)) = F(s)$

Then $L(f(t-a)\,H(t-a)) = e^{-sa} L\{f(t)\}$

$$= e^{-sa} F(s).$$

Where $H(t-a) = 0, \quad t < a$

$$= 1, \quad t \geq a.$$

Proof.

$$L(f(t-a)\,H(t-a)) = \int_0^\infty e^{-st} f(t-a) H(t-a)\, dt$$

$$= \int_0^a e^{-st}\, 0\, dt + \int_a^\infty e^{-st} f(t-a)\, dt$$

$$= \int_0^\infty e^{-s(a+x)} f(x)\, dx.$$

Let $t - a = x$

$dt = dx$

t	a	∞
x	0	∞

$$= \int_0^\infty e^{-as} e^{-sx} f(x)\, dx$$

$$= e^{-as} \int_0^\infty e^{-sx} f(x)\, dx$$

$$= e^{-as} F(s)$$

Worked Out Examples

~~Find Laplace transformation~~

1. Find $L\{e^{3t} \sin 4t\}$

Soln: $L(\sin 4t)$

$$= \frac{4}{s^2 + 4^2}$$

$$= \frac{4}{s^2+16}$$

$$= F(s), \text{ say}$$

$\therefore L(e^{3t} \sin 4t)$

$$= F(s-3) \quad \text{[By first shifting theorem]}$$

$$= \frac{4}{(s-3)^2+16}$$

$$= \frac{4}{s^2-6s+9+16}$$

$$= \frac{4}{s^2-6s+25}$$

2. Find the Laplace transformation of $f(t) = e^{-t}[2 - H(t-2)]$

$\boxed{\text{Soln}}$: $L(e^{-t})$

$$= \frac{1}{s-(-1)}$$

$$= \frac{1}{s+1}$$

$$= F(s), \text{ say}$$

$L\left(e^{-t}(2-H(t-2))\right)$

$= L\{2e^{-t} - e^{-t}H(t-2)\}$

$= 2L\{e^{-t}\} - L\{e^{-t}H(t-2)\}$ [As L is linear]

$= \frac{2}{s+1} - L\{e^{-(t-2)-2} \cdot H(t-2)\}$

$= \frac{2}{s+1} - L\left\{\frac{e^{-(t-2)}}{e^2} \cdot H(t-2)\right\}$

$= \frac{2}{s+1} - \frac{1}{e^2}L\{e^{-(t-2)} \cdot H(t-2)\}$ [As L is linear]

$= \frac{2}{s+1} - \frac{1}{e^2}e^{-2s}F(s)$ [By second shifting theorem]

$= \frac{2}{s+1} - \frac{1}{e^2}e^{-2s} \cdot \frac{1}{s+1}$

$= \frac{1}{s+1}\left[2 - \frac{e^{-2s}}{e^2}\right]$