B.Sc. II (Math (Hons))

Study Material

Paper – 4

Topic : Laplace Transfarmation

Material Sl. no – 7

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Laplace Transformation

Laplace Transformation of nth order derivatives

If $f(t), f'(t), f''(t), \ldots, f^{n-1}(t)$ are continuous for all $t \geq 0$ and of exponential order as $t \to \infty$ and if $f^{n}(t)$ is piecewise continuous for all $t \geq 0$ and of exponential order as $t \to \infty$, then,

$$L(f^{n}(t)) = s^{n} L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - f^{n-1}(0)$$

Proof.

From the previous the (i.e. Laplace Transformation of derivatives) we have

$$L(f'(t)) = s L(f(t)) - f(0)$$

Let $f'(t) = g(t)$ (say).

$$\therefore L(f''(t)) = L(g'(t))$$

$$= s L(g(t)) - g(0)$$

$= sL(f'(t)) - f'(0)$

$= s[sL(f(t)) - f(0)] - f'(0)$

$L(f''(t)) = s^2 L(f(t)) - sf(0) - f'(0)$

Again let $h(t) = f''(t)$

$\therefore L(f'''(t)) = L(h'(t))$

$= sL(h(t)) - h(0)$

$= s[L(f''(t))] - f''(0)$

$= s[s^2 L(f(t)) - sf(0) - f'(0)] - f''(0)$

$= s^3 L(f(t)) - s^2 f(0) - sf'(0) - f''(0)$

Continuing in this way, we get

$L(f^n(t)) = s^n L(f(t)) - s^{n-1} f(0) - s^{n-2} f'(0) - \ldots - f^{n-1}(0)$

This completes the proof.

$\boxed{\text{Laplace Transformation of integrals.}}$

If $f(t)$ is continuous for all $t \geq 0$ and of exponential order as $t \to \infty$, then

$$L\left(\int_0^t f(t)\,dt\right) = \frac{1}{s} L\{f(t)\}.$$

$\boxed{\text{Proof}}$

Let $g(t) = \int_0^t f(x)\,dx$.

Then clearly $g(0) = 0$

and $g'(t) = f(t)$, by fundamental theorem of calculus.

$\cancel{L\left(\int_0^t f(x)\,dx\right) = L(g(t))}$

Now $L(g'(t)) = s\,L(g(t)) - g(0)$

$= s\,L(g(t))$ [As $g(0) = 0$]

$\therefore L(g(t)) = \frac{1}{s} L(g'(t))$

Now, $L\left(\int_0^t f(x)\,dx\right) = L(g(t))$

$= \frac{1}{s} L(g'(t))$

$= \frac{1}{s} L(f(t))$

Consequently

$$\boxed{L\left(\int_0^t f(t)\,dt\right) = \frac{1}{s} L(f(t))}$$