B.Sc. II (Math (Hons))

Study Material

Paper – 4

Topic : Application of Laplace Transformation

Material Sl. no – 3

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Application of Laplace Transformation

Solution of simultaneous ODE

Example Solve the D.E.

$$(D-2)x + 3y = 0$$
$$2x + (D-1)y = 0, \quad x(0) = 8, \; y(0) = 3$$

Soln: First we rewrite the give D.E

$$x' - 2x + 3y = 0$$
$$2x + y' - y = 0$$

Applying Laplace transformation on both sides of the above two equations we get

$$L(x') - L(2x) + L(3y) = 0 \quad \text{---} \; (i)$$
$$2L(x) + L(y') - L(y) = 0 \quad \text{---} \; (ii)$$

From (i) we get

$$sL(x) - x(0) - 2L(x) + 3L(y) = 0$$
$$\text{or, } (s-2)L(x) + 3L(y) = 8 \quad \text{---} \; (iii)$$

Again from (ii) we get

$$2L(x) + sL(y) - y(0) - L(y) = 0$$
$$\text{or, } 2L(x) + (s-1)L(y) = 3 \quad \text{---} \; (iv)$$

(iii) $\times 2$ $-$ (iv) $(s-2)$ gives,

$$6L(y) - (s-1)(s-2)L(y) = 16 - 3(s-2)$$
$$\text{or, } L(y) \cdot [(s-1)(s-2) - 6] = 3(s-2) - 16$$

$$\text{or, } L(y)\left[s^2-3s+2-6\right] = 3s-6-16$$

$$\text{or, } L(y)\left[s^2-3s-4\right] = 3s-22$$

$$\text{or, } L(y) = \frac{3s-22}{s^2-4s+s-4}$$

$$y = L^{-1}\left(\frac{3s-22}{(s-4)(s+1)}\right) \longrightarrow (v)$$

Let $\frac{3s-22}{(s-4)(s+1)} = \frac{A}{s-4} + \frac{B}{s+1}$

$$= \frac{A(s+1)+B(s-4)}{(s-4)(s+1)}$$

$$\text{or, } 3s-22 = A(s+1)+B(s-4)$$

Putting $s=-1$, we get, $-25 = -5B$

$$\text{or, } B = 5$$

Again putting $s=4$, we get, $-10 = 5A$

$$\text{or, } A = -2$$

So from (v) we get

$$y = L^{-1}\left(\frac{-2}{s-4}\right) + L^{-1}\left(\frac{5}{s+1}\right)$$

$$y = -2e^{4t} + 5e^{-t}$$

Again Putting the value of $L(y)$ in (iii) we get

$$(s-2)L(x) + 3\cdot\frac{3s-22}{s^2-3s-4} = 8$$

$$\text{or, } (s-2)L(x) = \frac{8s^2-21s-32-9s+66}{s^2-3s-4}$$

$$\text{or, } (s-2)L(x) = \frac{8s^2 - 33s + 34}{(s-4)(s+1)}$$

$$= \frac{8s^2 - 16s - 17s + 34}{(s-4)(s+1)}$$

$$= \frac{8s(s-2) - 17(s-2)}{(s-4)(s+1)}$$

$$\text{or, } L(x) = \frac{8s - 17}{(s-4)(s+1)}$$

$$\text{or, } x = L^{-1}\left(\frac{8s-17}{(s-4)(s+1)}\right) \quad \text{— (VI)}$$

Let $\frac{8s-17}{(s-4)(s+1)} = \frac{C}{s-4} + \frac{D}{s+1}$

$$\text{or, } 8s - 17 = (s+1)C + (s-4)D$$

Putting $s = -1$, we get

$$-25 = -5D \quad \text{or, } D = 5$$

Again putting $s = 4$ we get

$$15 = 5C \quad \text{or, } C = 3$$

So from (VI) we have,

$$x = L^{-1}\left(\frac{3}{s-4}\right) + L^{-1}\left(\frac{5}{s+1}\right)$$

$$x = 3e^{4t} + 5e^{-t}$$

Hence $x = 3e^{4t} + 5e^{-t}$

& $y = 5e^{-t} - 2e^{4t}$ is the required solution