Study Material

B.Sc. II (Math (Hons))

Paper - 4

Topic: Inverse Laplace Transformation

Material Sl. no. - 1.

Prepared by: Rajat Subhra das

~~Assistant~~ Professor

Dept. L.K.V.D. College, Tajpur

Email: srajat das 10@gmail.com

Inverse Laplace Transformation

Definition

If $L(f(t)) = F(s)$, then $f(t)$ is said to be inverse Laplace transformation of $F(s)$. and Written as $L^{-1}(F(s)) = f(t)$

Inverse Laplace transformations of important functions

1. $L(K) = \frac{K}{s}$, i.e. $L^{-1}\left(\frac{K}{s}\right) = K$

2. $L^{-1}\left(\frac{1}{s-a}\right) = e^{at}$

3. $L^{-1}\left(\frac{1}{s^2+a^2}\right) = \frac{1}{a}\sin at$

4. $L^{-1}\left(\frac{s}{s^2+a^2}\right) = \cos at$

5. $L^{-1}\left(\frac{1}{s^2-a^2}\right) = \frac{1}{a}\sinh at$

6. $L^{-1}\left(\frac{s}{s^2-a^2}\right) = \cosh at$

7. $L^{-1}\left(\frac{n!}{s^{n+1}}\right) = t^n$.

Linearity

Inverse Laplace transformation is linear.

i.e if $L^{-1}(F(s)) = f(t)$ and $L^{-1}(G(s)) = g(t)$

Then $L^{-1}(aF(s) + bG(s)) = aL^{-1}(F(s)) + bL^{-1}(G(s))$

Where a & b are constant.

Proof

We already proved that Laplace-transformation is linear.

i.e $L\{af(t)+bg(t)\} = aL\{f(t)\} + bL\{g(t)\}$

$= aF(s) + bG(s)$

$\Rightarrow af(t) + bg(t) = L^{-1}\{aF(s) + bG(s)\}$

$\Rightarrow L^{-1}\{aF(s) + bG(s)\} = af(t) + bg(t)$

$= aL^{-1}\{F(s)\} + bL^{-1}\{G(s)\}$

This shows that L^{-1} is linear.

Problems

Find. $L^{-1}\left\{\frac{1}{3p-7}\right\}$

Soln:

$$L^{-1}\left(\frac{1}{3s-7}\right)$$

$$= L^{-1}\left(\frac{1}{3(s-7/3)}\right)$$

$$= \frac{1}{3}L^{-1}\left(\frac{1}{s-7/3}\right)$$

$$= \frac{1}{3}e^{7/3t}.$$

2. Find $L^{-1}\left(\frac{2s+7}{4s^2+9}\right)$

Soln

$$L^{-1}\left(\frac{2s+7}{4s^2+9}\right)$$

$$= L^{-1}\left(\frac{2s}{4s^2+9} + \frac{7}{4s^2+9}\right)$$

$$= 2L^{-1}\left(\frac{s}{4s^2+9}\right) + 7L^{-1}\left(\frac{1}{4s^2+9}\right)$$

$$= 2L^{-1}\left(\frac{s}{4(s^2+(3/2)^2)}\right) + 7L^{-1}\left(\frac{1}{4(s^2+(3/2)^2)}\right)$$

$$= \frac{1}{2}L^{-1}\left(\frac{s}{s^2+(3/2)^2}\right) + \frac{7}{4}L^{-1}\left(\frac{1}{s^2+(3/2)^2}\right)$$

$$= \frac{1}{2}\cos(3/2)t + \frac{7}{4}\cdot\frac{1}{3/2}\cdot\sin(3/2)t$$

$$= \frac{1}{2}\cos\frac{3}{2}t + \frac{7}{6}\sin(3/2t)$$