Study Material

B.Sc. II (Math (Hons))

Paper - 4

Topic: Inverse Laplace Transformation

Material Sl. no. - 3

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Inverse Laplace Transformation

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Division by s

$$L^{-1}\left(\frac{F(s)}{s}\right) = \int_0^t f(t)\,dt$$

Soln

Let $g(t) = \int_0^t f(x)\,dx$

$\therefore$ $g'(t) = f(t)$ and $g(0) = 0$

We know that

$$L(g'(t)) = sL(g(t)) - g(0)$$

As $g(0) = 0$.

So $L(g'(t)) = sL(g(t))$

or, $L(f(t)) = sL(g(t))$

or, $F(s) = sL(g(t))$

or, $\frac{1}{s}F(s) = L(g(t))$

or, $g(t) = L^{-1}\left(\frac{1}{s}f(s)\right)$

or, $\int_0^t f(x)\,dx = L^{-1}\left(\frac{1}{s}f(s)\right)$

or, $\int_0^t f(t)\,dt = L^{-1}\left(\frac{1}{s}f(s)\right)$

Worked out Examples.

1. Find $L^{-1}\left\{\frac{1}{(s-a)^4}\right\}$

Soln $L^{-1}\left\{\frac{1}{(s-a)^4}\right\}$

$= e^{a(at)} L^{-1}\left(\frac{1}{s^4}\right)$ [By shifting

$= e^{at} \frac{t^{4-1}}{(4-1)!}$

$= e^{at} \frac{t^3}{3!}$

$= \frac{t^3 e^{at}}{6}$

2. Find $L^{-1}\left(\frac{s+2}{s^2(s+3)}\right)$

Soln. Let $\frac{s+2}{s^2(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3}$

or, $\frac{s+2}{s^2(s+3)} = \frac{s(s+3)A + (s+3)B + s^2C}{s^2(s+3)}$

or, $s+2 = s(s+3)A + (s+3)B + s^2C$

Putting $s=0$ we get

$2 = 3B$ or, $B = 2/3$

Putting $s = -3$ we get

$-1 = 9C$ or, $C = -1/9$

Lastly putting $s = 1$, we get

$3 = 4A + 4B + C$

or, $4A = 3 - 4B - C$

$= 3 - \frac{8}{3} + \frac{1}{9}$

$= \frac{27 - 24 + 1}{9}$

$= 4/9$

$\therefore A = 1/9$

So $\frac{s+2}{s^2(s+3)} = \frac{1}{9}\cdot\frac{1}{s} + \frac{2}{3}\cdot\frac{1}{s^2} - \frac{1}{9(s+3)}$

Therefore

$$L^{-1}\left(\frac{s+2}{s^2(s+3)}\right) = \frac{1}{9}L^{-1}\left(\frac{1}{s}\right) + \frac{2}{3}L^{-1}\left(\frac{1}{s^2}\right) - \frac{1}{9}L^{-1}\left(\frac{1}{s+3}\right)$$

[As L^{-1} is linear]

$$= \frac{1}{9}\cdot 1 + \frac{2}{3}t - \frac{1}{9}e^{-3t}L^{-1}\left(\frac{1}{s}\right)$$

$$= \frac{1}{9} + \frac{2t}{3} - \frac{1}{9}e^{-3t}\cdot 1$$

$$= \frac{1 + 6t - e^{-3t}}{9}$$