Study Material for

B. Sc. II (Math (Sub/Gen))

Topic: Differential Equation

Sub-topic: D.E. of Ist order and Ist degree

Material Sl. no — 1

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Differential Equation of First Order and First Degree

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Solvable by Separation of Variables

D.E. $\quad M\,dx + N\,dy = 0$

where M is functions x & y or constant

N "

If we can rearrang the given D.E. in the form $f(x)\,dx + g(y)\,dy = 0$, where $f(x)$ is a function of x and $g(y)$ is a function of y, then the rearranged equation is said to have separated variables.

Its general soln is:

$$\int f(x)\,dx + \int g(y)\,dy = c$$

where c is an arbitrary constant.

Worked out Examples

1. Solve: $(1+x)\,dy + (1+y)\,dx = 0$

Soln:

$$(1+x)\,dy + (1+y)\,dx = 0$$

or, $$(1+x)\,dy = -(1+y)\,dx$$

or, $$\frac{dy}{1+y} = -\frac{dx}{1+x}$$

or, $\frac{dx}{1+x} + \frac{dy}{1+y} = 0$

Hence the general soln is

$$\int \frac{dx}{1+x} + \int \frac{dy}{1+y} = c_1$$

or, $\log|(1+x)| + \log|(1+y)| = \log C$

or, $\log|(1+x)(1+y)| = \log C$ [where $\log C = c_1$]

which is the required soln

(2) Solve $\frac{dy}{dx} = e^{x+y} + x^2 e^y$

Soln:

$$\frac{dy}{dx} = e^{x+y} + x^2 e^y$$

or, $\frac{dy}{dx} = e^x . e^y + x^2 e^y$

or, $\frac{dy}{dx} = e^y (e^x + x^2)$

or, $\frac{dy}{e^{-y}} = (e^x + x^2)\, dx$

or, $(e^x + x^2)\, dx - e^{-y}\, dy = 0$

Hence the general soln is

$$\int (e^x + x^2)\, dx - \int e^{-y}\, dy = c$$

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or, $e^{x} + \frac{x^3}{3} + e^{-y} = c$, where c is an integrating constant

ans,

③ Solve: $y + \frac{dy}{dx}(1+x^2)\tan^{-1}x = 0$

Soln:. $y + \frac{dy}{dx}(1+x^2)\tan^{-1}x = 0$

or, $\frac{dy}{dx}(1+x^2)\tan^{-1}x = -y$

or, $\frac{dy}{y} = -\frac{dx}{(1+x^2)\tan^{-1}x}$

or, $\frac{dx}{(1+x^2)\tan^{-1}x} + \frac{dy}{y} = 0$.

Hence the general soln is.

$$\int \frac{dx}{(1+x^2)\tan^{-1}x} + \int \frac{dy}{y} = c,$$

or, $\int \frac{dz}{z} + \log|y| = c,$ Let $\tan^{-1}x = z$

$\frac{1}{1+x^2}dx = dz$

or, $\log|z| + \log|y| = c,$

or, $\log|\tan^{-1}x| + \log|y| = c$., ~~where~~

where c is an integrating constant.

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Homework for Students :

Solve the following differential equation.

(i) $$\frac{dy}{dx} = \frac{xy^2 - x}{yx^2 + y}$$

(ii) $$\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$$

(iii) $$\log\left(\frac{dy}{dx}\right) = ax + by$$

(iv) $$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

(v) $$\frac{dy}{dx} + \frac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}} = 0$$

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