Study Material for

B.Sc. II (Math (Sub/Gen))

Topic: Differential Equation

Sub-topic: D.E. of Ist order and Ist degree

Material Sl. no — 2

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Differential Equation of First order and first degree

Equation formable into seperated of variable form.

Solve $\frac{dy}{dx} = \sin(x+y)$

Soln:

$$\frac{dy}{dx} = \sin(x+y)$$

Let $x+y = z$

or, $1 + \frac{dy}{dx} = \frac{dz}{dx}$

or, $\frac{dy}{dx} = \frac{dz}{dx} - 1$

So, $\frac{dz}{dx} - 1 = \sin z$

or, $\frac{dz}{dx} = 1 + \sin z.$

or, $\frac{dz}{1+\sin z} = dx$

or, $\frac{dz}{1+\sin z} - dx = 0$

Hence the general soln is.

$$\int \frac{dz}{1+\sin z} - \int dx = c_1$$

or, $\int \frac{1-\sin z}{(1+\sin z)(1-\sin z)} dz - \int dx = c_1$

or, $\int \frac{1-\sin z}{\cos^2 z}\, dz - x = c_1$

or, $\int \frac{1}{\cos^2 z}\, dz - \int \frac{\sin z}{\cos^2 z}\, dz - x = c_1$

or, $\int \sec^2 z\, dz - \int \sec z \tan z\, dz - x = c_1$

or, $\tan z - \sec z - x = c_1$

or, $\tan(x+y) - \sec(x+y) - x = c_1$,

where c_1 is an integrating constant.

② Solve: $(x-y)^2 \frac{dy}{dx} = m^2$

Soln: $(x-y)^2 \frac{dy}{dx} = m^2$.

Let $x - y = z$

or, $1 - \frac{dy}{dx} = \frac{dz}{dx}$

or, $\frac{dy}{dx} = 1 - \frac{dz}{dx}$

So, $z^2\left(1 - \frac{dz}{dx}\right) = m^2$

or, $z^2 - z^2\frac{dz}{dx} = m^2$

or, $-z^2\frac{dz}{dx} = m^2 - z^2$

or, $z^2\frac{dz}{dx} = z^2 - m^2$

or, $\frac{z^2}{z^2-m^2}\,dz = dx$

or, $\frac{z^2}{z^2-m^2}\,dz - dx = 0$

Integrating we get,

$$\int \frac{z^2}{z^2-m^2}\,dz = \int dx$$

or, $\int \frac{z^2-m^2+m^2}{z^2-m^2}\,dz = x + C_1$

or, $\int dz + \int \frac{m^2}{z^2-m^2}\,dz = x + C_1$

or, $z + \frac{m^2}{2m}\log\left|\frac{z-m}{z+m}\right| = x + C$

or, $x - y + \frac{m}{2}\log\left|\frac{x-y-m}{x-y+m}\right| = x + C$

or, $\frac{m}{2}\log\left|\frac{x-y-m}{x-y+m}\right| = y + C$

~~③ Soln: $(x-y)^2 \frac{dy}{dx^2}$~~

③ Solve: $\frac{dy}{dx} + 2xy = x^2 + y^2$

Soln:

$$\frac{dy}{dx} = x^2 + y^2 - 2xy$$

or, $\frac{dy}{dx} = (x-y)^2$

Let $x - y = z$

$1 - \frac{dy}{dx} = \frac{dz}{dx}$

or, $\frac{dy}{dx} = 1 - \frac{dz}{dx}$

or, $1 - \frac{dz}{dx} = z^2$

or, $-\frac{dz}{dx} = z^2 - 1$

or, $\frac{dz}{z^2-1} = -dx$

on Integrating we get

$$\int \frac{dz}{z^2-1} = -\int dx$$

or, $\frac{1}{2}\log\left|\frac{z-1}{z+1}\right| = -x + C$

or, $\frac{1}{2}\log\left|\frac{x-y-1}{x-y+1}\right| + x = C$, where C is an integrating constant

Homework for Students.

1. Solve the following D.E.

(i) $\frac{dy}{dx} = \sqrt{y - x}$

(ii) $(x+y)^2 \frac{dy}{dx} = m^2$

(iii) $\cos^{-1}\left(\frac{dy}{dx}\right) = x + y$

(iv) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$