SUBJECT - CHEMISTRY

CLASS - B.Sc (Hons) PART - III

PAPER - V

TOPIC - Degree of degeneracy

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Q Determine the degree of degeneracy of the energy level $\frac{17h^2}{8ma^2}$ for a particle in a cubical box.

The energy of a particle in a cubical box is given by -

$$E = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$$

from question

$$\frac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right) = 17 \cdot \frac{h^2}{8mL^2}$$

$$\text{or} \quad n_x^2 + n_y^2 + n_z^2 = 17$$

possible arrangements for the sum of squared terms will be 17, are given as —

n_x	n_y	n_z
3	2	2
2	3	2
2	2	3

As three sets of quantum numbers give the same energy hence the degree of degeneracy is three.

Q List the restrictions on a wave function and explain how These lead to quantisation.

Ans A wave function describing a free particle inside a one dimensional box is given by -

$$\psi = A \sin \sqrt{\frac{2mE}{\hbar}} \cdot x + B \cos \sqrt{\frac{2mE}{\hbar}} \cdot x$$

This wave function does not tell us much information because it offers no restriction on the energy E of the particle. Actually a free particle moving along the x-axis can have energy i.e a free particle has a Continuous set of energy levels.

Quantisation, however, can take place when -

1 The particle is not entirely free
2 The particle be constrained to remain in a box.
3 The particle be influenced by the walls.
4 Since the potential is very high at the walls of the box, the probability of finding the particle at the walls is zero i.e $|\psi|^2 = 0$ at the walls. The Schrodinger equation is given by -

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE}{\hbar^2} \psi = 0 \quad \text{--- (1)}$$

$$\therefore \quad \psi = A \sin kx + B \cos kx \quad \cdot\cdot (2)$$

Conditions are (i) $\psi(0) = 0$ (ii) $\psi(a) = 0$

putting the first Condition in equation (2) we get -

$$0 = B$$

putting the second Condition in equation (2) we get -

$$0 = A \sin ka$$

$$A \neq 0$$

~~So $\sin ka$ = $\sin nx$~~

So $\sin ka = \sin n\pi$

or $ka = n\pi$