SUBJECT - CHEMISTRY

CLASS - B.Sc (Hons) PART - III

PAPER - V

TOPIC - EIGEN VALUE & EIGEN FUNCTION

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Q. what are eigen values and eigen functions?

Ans. $y = f(x)$ i.e y is a function of x because the change in the value of x produces the change in the value of y. If an operator ($\hat{A}$) operates on a well behaved function (ψ) to give the Same function (ψ) but multiplied by a Constant (E), then the Constant (E) is Called eigen value and the function (ψ) as eigen function and The equation as eigen value equation. So, a typical eigen value equation is given by -

$$\hat{A}\psi = E\psi$$

where $\hat{A}$ is an operator, E is the eigen value and ψ is the wave function. Eigen value is Just a real number. The linear momentum operator along x-axis is $-i\hbar \cdot \frac{d}{dx}$. Hence the eigen value equation is given as

$$-i\hbar \frac{d\psi}{dx} = E\psi$$

where E is eigen value for linear momentum along x-axis ($\hat{P}_x$).

we see that an operator is the Seed. eigen function is the soil and eigen value is the crop. So the kind of crop depends only on the nature of Seed used on the Soil. Similarly the kind of eigen value depends on the kind of operator used.

Q Show that the function $\psi = 8e^{4x}$ is an eigen function of operator $\frac{d}{dx}$. What is eigen value?

Ans $\frac{d}{dx}(8e^{4x}) = 8.4e^{4x} = 4.8e^{4x}$

Thus $8e^{4x}$ is an eigen function of the operator $\frac{d}{dx}$ and its eigen value is 4.

Q Show that the function $\psi = e^{-mx}$ is the eigen function of Operator $\frac{d}{dx}$

Ans If we operate a function $\psi = e^{-mx}$ by an operator $\frac{d}{dx}$

we get $\frac{d}{dx} e^{-mx} = -m.e^{-mx} = -m.\psi$ cf. $\hat{A}\psi = E\psi$

Hence ψ is a wave function, E is eigen value and $\hat{A}$ is an operator.

Q. Show that the function $\psi = \sin 2x$ is not the eigen function of operator $\frac{d}{dx}$ but of $\frac{d^2}{dx^2}$.

Ans If we operate a function $\psi = \sin 2x$ by operators $\frac{d}{dx}$ and $\frac{d^2}{dx^2}$.

we get –

$$\frac{d}{dx}(\sin 2x) = 2\cos 2x \quad \text{--- (I)}$$

$\left[\because y = \sin mx \therefore \frac{dy}{dx} = m\cos mx\right]$

$$\frac{d^2}{dx^2}(\sin 2x) = -4(\sin 2x) = -4\psi \quad \text{--- (II)}$$

$\left[\because y = \cos mx \therefore \frac{dy}{dx} = -m\sin mx\right]$

Eq. (II) gives the eigen value -4 and the original function after operation. Hence $\psi = \sin 2x$ is not the eigen function of $\frac{d}{dx}$ but of $\frac{d^2}{dx^2}$.