

Heisenberg Uncertainty principle: "It is impossible to determine precisely and simultaneously the momentum and position of the electron in an atom." If the position of electron is known, then its velocity (and momentum) is uncertain; if its velocity is known, then its position is uncertain. Mathematically, Heisenberg's uncertainty principle is expressed as -

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi} \text{ and } \frac{\hbar}{2} = 5.27 \times 10^{-35} \text{ Js,}$$

and  $h = \text{Planck's constant } (6.627 \times 10^{-34} \text{ Js})$

$\Delta x$  and  $\Delta p$  represent the respective uncertainties in their position and momentum. In other words, the product of uncertainties in position ( $\Delta x$ ) and momentum ( $\Delta p_x$ ) along the  $x$  direction of a body is equal to or greater than  $\frac{\hbar}{2}$  but shall not be less than  $\frac{\hbar}{2}$ . Smaller the  $\Delta x$  value, larger will be  $\Delta p$  value i.e. the more exactly the position is determined the less accurately, momentum be measured simultaneously and vice-versa.

$$\text{Again } \Delta x \cdot m \Delta v \sim \frac{\hbar}{2} \Rightarrow \Delta x \propto \frac{1}{\Delta v}$$

Slower the speed, more localised is the position of the particle.

Uncertainty i.e.  $\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$  is a fundamental limit of nature and holds for a pair of conjugate quantities.

Now the uncertainty principle is extended to cover any two conjugate quantities like position and momentum, angular position and angular momentum, time and energy etc. If the dimensions of the product of two variables are the dimensions of  $h$  (Planck's constant) i.e.  $\text{kgm}^2/\text{s}$  or Js, then they are called 'conjugate quantities' or action. Hence the uncertainty relation of energy and time is given as

$$\Delta E \cdot \Delta t \sim \frac{\hbar}{2} \Rightarrow \Delta E \propto \frac{1}{\Delta t}$$

Hence shorter the life, greater is the energy of the particle.

The distinction between micro and macro particles is made on the basis of 'action'. If the action  $\leq \hbar/2$ , the system is a micro system i.e. quantum mechanical system otherwise a macro system i.e. classical system.

Q. Determine the uncertainty in the velocity of electron when we want to locate it within a range of  $0.5 \text{ \AA}$ .

Ans According to Heisenberg uncertainty principle

$$\Delta x \cdot \Delta p \sim \frac{\hbar}{2} \quad \text{or} \quad \Delta x \cdot m \Delta v \sim \frac{\hbar}{2}$$

$$\therefore \Delta v \sim \frac{\hbar}{2 \cdot m \cdot \Delta x} \sim \frac{5.27 \times 10^{-35}}{9.1 \times 10^{-31} \times 0.5 \times 10^{-10}} \sim 1.17 \times 10^6 \text{ m/sec.}$$