

Q Explain the meaning of quantum degeneracy.

Ans Quantum degeneracy:- The same energy states with different eigen functions are called degenerate states. This phenomenon is called quantum degeneracy. The complete wave function Ψ for a particle (electron) in a cubical box of length L is given by

$$\Psi_{n_x n_y n_z} = \sqrt{\frac{8}{L^3}} \cdot \sin\left(\frac{n_x \pi}{L}\right) \cdot x \cdot \sin\left(\frac{n_y \pi}{L}\right) \cdot y \cdot \sin\left(\frac{n_z \pi}{L}\right) \cdot z \quad (1)$$

And its total energy (E) is given by -

$$E_{n_x n_y n_z} = \frac{h^2}{8mL^2} (n_x^2 + n_y^2 + n_z^2) \quad (2)$$

Where n_x , n_y and n_z are quantum numbers along x , y and z axes and are integers except zero.

For states having quantum numbers $(2, 1, 1)$, $(1, 2, 1)$ and $(1, 1, 2)$ eigen functions and their energies are given below -

States ($n_x n_y n_z$)	$\Psi_{n_x n_y n_z}$	$E_{n_x n_y n_z}$
2, 1, 1	$\Psi_{211} = \sqrt{\frac{8}{L^3}} \sin\left(\frac{2\pi}{L}\right) \cdot x \sin\left(\frac{\pi}{L}\right) \cdot y \sin\left(\frac{\pi}{L}\right) \cdot z$	$E_{211} = \frac{6h^2}{8mL^2}$
1, 2, 1	$\Psi_{121} = \sqrt{\frac{8}{L^3}} \sin\left(\frac{\pi}{L}\right) \cdot x \sin\left(\frac{2\pi}{L}\right) \cdot y \sin\left(\frac{\pi}{L}\right) \cdot z$	$E_{121} = \frac{6h^2}{8mL^2}$
1, 1, 2	$\Psi_{112} = \sqrt{\frac{8}{L^3}} \sin\left(\frac{\pi}{L}\right) \cdot x \sin\left(\frac{\pi}{L}\right) \cdot y \sin\left(\frac{2\pi}{L}\right) \cdot z$	$E_{112} = \frac{6h^2}{8mL^2}$

where $2^2 + 1^2 + 1^2 = 1^2 + 2^2 + 1^2 = 1^2 + 1^2 + 2^2 = 6$

We see that these three states have the same energy. So the energy level corresponding to energy $6h^2/8ml^2$ is three fold or triply degenerate. These np_z , np_x and np_y orbitals for which $m = +1, 0, -1$ constitute three independent states as their eigen functions are different but they give the same energy. Hence p-orbitals are triply degenerate. Similarly d and f-orbitals have five and seven fold degeneracy respectively in absence of any applied field.